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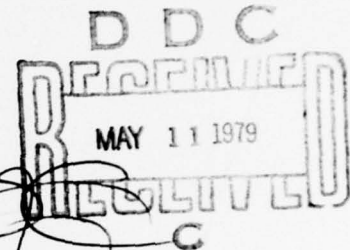
ANALYSIS OF ARBITRARILY SHAPED WIRE ANTENNAS
RADIATING OVER A LOSSY HALF-SPACE

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INTERIM TECHNICAL REPORT

P. PARHAMI
R. MITTRA

MARCH 1979



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ELECTROMAGNETICS LABORATORY
DEPARTMENT OF ELECTRICAL ENGINEERING
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ANALYSIS OF ARBITRARILY SHAPED WIRE ANTENNAS
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The problem of an arbitrarily oriented current element over a lossy half-space is analyzed in this work. Various numerical approaches are developed and discussed for evaluating the infinite Sommerfeld integrals appearing in the vector potential expressions. In particular, a technique based on the steepest descent path integration is introduced for exact evaluation of these integrals, and, as an efficient alternative, an additional technique is developed based on approximating the well-behaved Fourier transform expressions of the Sommerfeld integrals. The latter technique has the merit of not requiring any time-consuming infinite integrations and, at the same time, is shown to yield accurate results for a wide range of parameters of practical interest. Finally, in this paper, the thin-wire antenna problem over a lossy half-space is analyzed via the method of moments and the current element solution techniques discussed earlier. Several antenna examples are included to demonstrate the effect of the lossy half-space on their input impedance and the far-field radiation patterns.

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1. INTRODUCTION

The conventional approach to analyzing antenna structures radiating in the presence of a lossy half-space involves repeated evaluation of the Sommerfeld integrals, which were originally introduced by Sommerfeld about 70 years ago [1] and appeared in the expressions for the vector potentials [2]. Since these infinite integrals are generally highly oscillatory and difficult to evaluate numerically, much attention has been focused in recent years on developing techniques for efficiently evaluating the Sommerfeld integrals without unduly sacrificing the accuracy [3] - [14]. Even though the latest reported procedures [7] - [12] require an order of magnitude less computing time than the earlier "brute-force" numerical integration techniques [3] - [5], the overall computing time severely limits the physical dimensions of the antenna structures being numerically analyzed. Brittingham et al. [10] have used an interpolation scheme on a precalculated grid of Sommerfeld integral values in order to analyze larger structures. However, this technique is only useful for fixed frequency and ground parameters, since a new grid is required each time any of these parameters are changed.

Considerable computing time is saved if one uses the first term in the asymptotic expansion of the Sommerfeld integrals, better known as the Fresnel's Reflection Coefficient Method (RCM). These approximations, which are expressed in a simple closed form, are valid only when the antenna structures and the observation points are sufficiently high above the ground. As expected, the RCM expressions have only been employed for analyzing antenna structures at the high end of the frequency spectrum [10], [12], [13].

An important contribution of this paper is the development of a novel technique which is computationally comparable to the RCM approximation, yet valid for a much wider range of parameters.

In Chapter 2, the complex vector potential expressions for an arbitrarily oriented electric-current element source over a lossy half-space are derived demonstrating that three of the vector potential components contain the troublesome Sommerfeld integrals. An efficient numerical technique, based on the Steepest Descent Path (SDP) integration, is presented in Chapter 3 for evaluating the exact Sommerfeld integral expressions, and the results are compared with the one- and two-term asymptotic expansions of these integrals. In Chapter 4, a unique procedure is developed in which the well-behaved Fourier transform representations of the Sommerfeld integrals are approximated such that the inverse transform is performed via a set of known exact identities. These approximate expressions are shown not only to closely follow the SDP integration results for a wide range of parameters of practical interest, but to be in a convenient form for numerical evaluation. Finally, in Chapter 5, several examples, which are solved via the method of moments in conjunction with the approximate expressions derived in Chapter 4, are presented of various antenna structures radiating over a lossy half-space.

2. ARBITRARY CURRENT ELEMENT RADIATING OVER LOSSY HALF-SPACE

The geometry of an arbitrarily oriented current element P_1 over a lossy half-space is depicted in Figure 1. Regions 1 and 2 are characterized by $(\epsilon_1 = \epsilon_{1r}\epsilon_0, \mu_1 = \mu_0)$ and $(\epsilon_2 = \epsilon_{2r}\epsilon_0, \mu_2 = \mu_0)$, respectively, where ϵ_0 and μ_0 are free-space parameters. In addition to the standard Cartesian coordinate system (x, y, z) , two spherical coordinate systems (r, θ_1, ϕ_1) and (r_2, θ_2, ϕ_2) are also defined in Figure 1 centered about the source point P_1 and its geometrical image point P_2 , respectively. Our objective is to determine the field radiated by P_1 at the observation point 0, in the presence of the lossy half-space (region 2).

2.1 Vector Potential Approach

Starting with Maxwell's equation and the suppressed time convention $\exp(j\omega t)$, viz.,

$$\nabla \times \vec{H} = j\omega\epsilon_0\epsilon_r \vec{E} + \vec{J} \quad (2.1a)$$

$$\nabla \times \vec{E} = -j\omega\mu_0 \vec{H} \quad , \quad (2.1b)$$

one may define the vector potential \vec{A} as

$$\vec{H} = j\omega\epsilon_0\epsilon_r \nabla \times \vec{A} \quad . \quad (2.2)$$

Introduction of a scalar potential Φ from

$$\nabla\Phi = \vec{E} - \omega^2\mu_0\epsilon_0\epsilon_r \vec{A} \quad (2.3)$$

and application of the Lorentz gauge

$$\nabla \cdot \vec{A} - \Phi = 0 \quad , \quad (2.4)$$

allows one to finally express Maxwell's equation as

$$(\nabla^2 + k^2)\vec{A} = -(j\omega\epsilon_0\epsilon_r)^{-1} \vec{J} \quad , \quad (2.5)$$

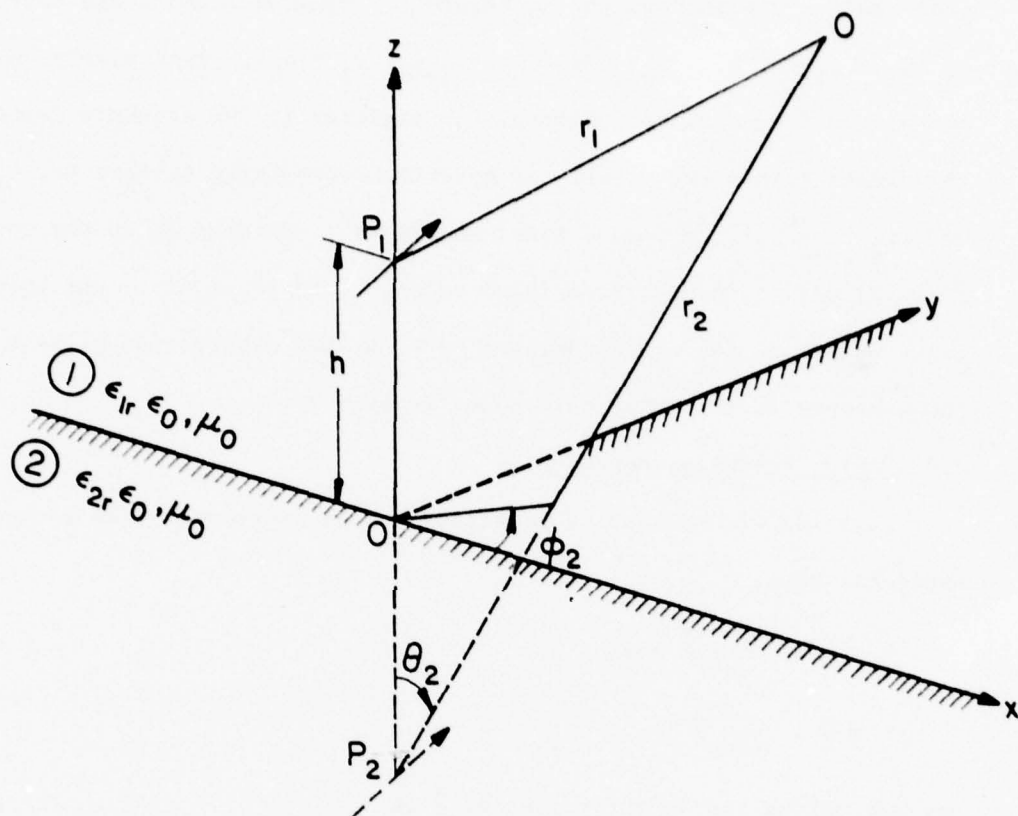


Figure 1. Geometry and the coordinate systems for the current element P_1 radiating over imperfect ground, where $\epsilon_{1r} = 1$ and $\epsilon_{2r} = \epsilon_g - j\sigma/(\omega\epsilon_0)$ have been assumed.

$$\vec{H} = j\omega\epsilon_0\epsilon_r \nabla \times \vec{\Pi} \quad (2.6)$$

$$\vec{E} = (\nabla\nabla \cdot + k^2)\vec{\Pi} \quad (2.7)$$

where $k^2 = \omega^2\mu_0\epsilon_0\epsilon_r$. The preceding results are general and valid for both regions 1 and 2. The boundary conditions needed to solve the vector differential Equation (2.5) can be obtained simply by enforcing the continuity of the tangential \vec{E} - and \vec{H} -field components at the interface.

As in most infinite-interface-type problems reported in the literature, the Fourier transform technique is employed for deriving the vector potential expressions. The two-dimensional Fourier transform pair is defined as

$$\tilde{\vec{\Pi}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{\Pi} \exp[-j(\alpha x + \beta y)] dx dy \quad (2.8a)$$

$$\vec{\Pi} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\vec{\Pi}} \exp[j(\alpha x + \beta y)] d\alpha d\beta \quad (2.8b)$$

Throughout this paper, \sim on top denotes the transformed quantity.

Without loss of generality, the problem of an arbitrarily oriented current source over lossy half-space (Figure 1) can be treated as two independent cases: first, the problem of a vertical current source oriented in the z -direction and, second, a horizontal source in the x -direction. These two cases are analyzed in detail in the following sections with subscripts v and h denoting the field quantities belonging to the vertical and the horizontal current sources, respectively. Once the vector potential expressions are derived, the \vec{E} - and \vec{H} -field components are directly obtained by transforming the general vector equations given in (2.6) and (2.7) into the following convenient matrix forms:

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} k^2 - \alpha^2 & -\alpha\beta & j\alpha \frac{\partial}{\partial z} \\ -\alpha\beta & k^2 - \beta^2 & j\beta \frac{\partial}{\partial z} \\ j\alpha \frac{\partial}{\partial z} & j\beta \frac{\partial}{\partial z} & (k^2 + \frac{\partial^2}{\partial z^2}) \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_x \\ \tilde{\Pi}_y \\ \tilde{\Pi}_z \end{bmatrix} \quad (2.9)$$

and

$$\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \\ \tilde{H}_z \end{bmatrix} = j\omega\epsilon_0\epsilon_r \begin{bmatrix} 0 & -\frac{\partial}{\partial z} & j\beta \\ \frac{\partial}{\partial z} & 0 & -j\alpha \\ -j\beta & j\alpha & 0 \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_x \\ \tilde{\Pi}_y \\ \tilde{\Pi}_z \end{bmatrix} \quad (2.10)$$

2.2 Vertical Current Element-Transform Domain Expressions

The current element is assumed to be in the z -direction (see Figure 1), located at height h over the lossy ground, and with a moment $I_v dz'$:

$$\vec{J}_v = \hat{z} I_v dz' \delta(x) \delta(y) \delta(z) \quad (2.11)$$

As has been verified in the literature [2], only the vertical component of the vector potential is needed to satisfy all of the boundary conditions, namely

$$\vec{\Pi}_{vi} = \hat{z} \Pi_{viz} \quad ; \quad i = 1, 2 \quad (2.12)$$

where $i = 1, 2$ indicates the region under consideration. Using the source condition in (2.11), one can express the general solution for the vector potential $\tilde{\Pi}_{viz}$ by satisfying both the wave Equation (2.5) and the radiation conditions as

$$\tilde{\Pi}_{v1z} = I_{v0} \exp[-j\gamma_1 |z - h|] / 2j\gamma_1 + A_1 \exp(-j\gamma_1 z) \quad ; \quad z \geq 0 \quad (2.13a)$$

$$\tilde{\Pi}_{v2z} = A_2 \exp(j\gamma_2 z) \quad ; \quad z \leq 0 \quad (2.13b)$$

where A_1 and A_2 are arbitrary constants and

$$I_{v0} = (j\omega\epsilon_0\epsilon_{1r})^{-1} I_v dz' \quad (2.14)$$

$$\gamma_i = [k_i^2 - \alpha^2 - \beta^2]^{1/2} ; \quad I_m(\gamma_i) \leq 0 ; \quad i = 1, 2 \quad (2.15)$$

$$k_i^2 = \omega^2 \mu_0 \epsilon_{1r} \epsilon_0 ; \quad i = 1, 2 \quad (2.16)$$

If the continuity of the tangential components of the E- and H-fields across the boundary is enforced, the following constraints on $\tilde{\Pi}_{viz}$ are obtained

$$\frac{\partial}{\partial z} \tilde{\Pi}_{v1z} \Big|_{z=0} = \frac{\partial}{\partial z} \tilde{\Pi}_{v2z} \Big|_{z=0} \quad (2.17a)$$

$$\tilde{\Pi}_{v1z} \Big|_{z=0} = \kappa \tilde{\Pi}_{v2z} \Big|_{z=0} \quad (2.17b)$$

where the complex constant κ is defined as

$$\kappa = \epsilon_{2r} / \epsilon_{1r} \quad (2.18)$$

Constants A_1 and A_2 , present in the vector potential expressions (2.13a) and (2.13b) can be determined by satisfying the boundary conditions (2.17a) and (2.17b):

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = I_{v0} \frac{1}{j(\kappa\gamma_1 + \gamma_2)} \begin{bmatrix} \frac{\kappa\gamma_1 - \gamma_2}{2\gamma_1} \\ 1 \end{bmatrix} \exp(-j\gamma_1 h) \quad (2.19)$$

If (2.19) is substituted back into (2.13a) and (2.13b), the complete vector potential expression in the transform domain can be written in the following form:

$$\tilde{\Pi}_{v1z} = \tilde{\Pi}_{v1z}^i + \tilde{\Pi}_{v1z}^r + \tilde{\Pi}_{v1z}^0 \quad (2.20)$$

where

$$\tilde{\Pi}_{v1z}^i = I_{v0} \exp[-j\gamma_1 |z - h|] / 2j\gamma_1 \quad (2.21a)$$

$$\tilde{\Pi}_{vlz}^r = -I_{v0} \exp[-j\gamma_1(z+h)]/2j\gamma_1 \quad (2.21b)$$

$${}_0\tilde{\Pi}_{vlz} = I_{v0} \frac{\kappa}{j(\kappa\gamma_1 + \gamma_2)} \exp[-j\gamma_1(z+h)] \quad (2.21c)$$

and

$$\tilde{\Pi}_{v2z} = I_{v0} \frac{1}{j(\kappa\gamma_1 + \gamma_2)} \exp(-j\gamma_1 h) \exp(j\gamma_2 z) \quad (2.22)$$

Note that the first two terms of $\tilde{\Pi}_{vlz}$, viz., $\tilde{\Pi}_{vlz}^i$ and $\tilde{\Pi}_{vlz}^r$, can be interpreted as the direct and the perfect ground reflection contributions to the fields for observation points in region 1. Thus, the remaining component ${}_0\tilde{\Pi}_{vlz}$ is simply the correction term to the perfect ground solution in the general lossy half-space problem.

2.3 Horizontal Current Element-Transform Domain Expressions

The current element is assumed to be in the x-direction (see Figure 1), located at height h over lossy ground and with a moment $I_h dx'$:

$$\vec{J}_h = \hat{x} I_h dx' \delta(x) \delta(y) \delta(z-h) \quad (2.23)$$

As expected, the horizontal current element solution above a lossy half-space is indeed more complicated than that for the vertical case, and two vector potential components are needed to obtain a complete solution [2], namely,

$$\vec{\Pi}_{hi} = \hat{x} \Pi_{hix} + \hat{z} \Pi_{hiz} \quad ; \quad i = 1, 2 \quad (2.24)$$

Using the source condition in (2.23), the general solutions for the vector potential components satisfying both the wave Equation (2.5) and the radiation condition can be expressed as

$$\begin{bmatrix} \tilde{\Pi}_{h1x} \\ \tilde{\Pi}_{h1z} \end{bmatrix} = \begin{bmatrix} I_{h0} \exp[-j\gamma_1|z-h|]/2j\gamma_1 \\ 0 \end{bmatrix} + \begin{bmatrix} B_{1x} \\ B_{1z} \end{bmatrix} \exp(-j\gamma_1 z) \quad ; \quad z \geq 0 \quad (2.25)$$

$$\begin{bmatrix} \tilde{\Pi}_{h2x} \\ \tilde{\Pi}_{h2z} \end{bmatrix} = \begin{bmatrix} B_{2x} \\ B_{2z} \end{bmatrix} \exp(j\gamma_2 z) \quad ; \quad z \leq 0 \quad (2.26)$$

where γ_i is defined in (2.15) and I_{h0} is assumed to be

$$I_{h0} = (j\omega\epsilon_0\epsilon_{lr})^{-1} I_h \, dx' \quad (2.27)$$

If the continuity of the tangential components of the E- and the H-fields across the boundary is enforced, the following constraints on $\tilde{\Pi}_{h1x}$ and $\tilde{\Pi}_{h1z}$ are obtained:

$$\begin{bmatrix} j\alpha & \frac{\partial}{\partial z} & -j\alpha & -\frac{\partial}{\partial z} \\ 1 & 0 & -\kappa & 0 \\ 0 & 1 & 0 & -\kappa \\ \frac{\partial}{\partial z} & 0 & -\kappa \frac{\partial}{\partial z} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_{h1x} \\ \tilde{\Pi}_{h1z} \\ \tilde{\Pi}_{h2x} \\ \tilde{\Pi}_{h2z} \end{bmatrix} = 0 \quad ; \quad z = 0 \quad (2.28)$$

The conditions in (2.28) are used to determine the constants B_{1x} and B_{1z} present in the general vector potential expressions given in (2.25) and (2.26):

$$\begin{bmatrix} B_{1x} \\ B_{1z} \end{bmatrix} = I_{h0} \begin{bmatrix} \frac{\gamma_1 - \gamma_2}{2j\gamma_1(\gamma_1 + \gamma_2)} \\ \frac{j\alpha(\gamma_1 - \gamma_2)}{k_1^2(\kappa\gamma_1 + \gamma_2)} \end{bmatrix} \exp(-j\gamma_1 h) \quad ; \quad z \geq 0 \quad (2.29a)$$

$$\begin{bmatrix} B_{2x} \\ B_{2z} \end{bmatrix} = I_{h0} \begin{bmatrix} \frac{1}{j\kappa(\gamma_1 + \gamma_2)} \\ \frac{j\alpha(\gamma_1 - \gamma_2)}{k_1^2(\kappa\gamma_1 + \gamma_2)} \end{bmatrix} \exp(-j\gamma_1 h) \quad ; \quad z \leq 0 \quad (2.29b)$$

If (2.29a) and (2.29b) are substituted back into (2.25) and (2.26), the complete transform domain vector potential expressions can be put into the following convenient form:

$$\tilde{\Pi}_{hlx} = \tilde{\Pi}_{hlx}^i + \tilde{\Pi}_{hlx}^r + \tilde{\Pi}_{hlx}^0 \quad (2.30)$$

where

$$\tilde{\Pi}_{hlx}^i = I_{h0} \exp [-j\gamma_1 |z - h|] / 2j\gamma_1 \quad (2.31a)$$

$$\tilde{\Pi}_{hlx}^r = -I_{h0} \exp [-j\gamma_1 (z + h)] / 2j\gamma_1 \quad (2.31b)$$

$$\tilde{\Pi}_{hlx}^0 = I_{h0} \frac{1}{j(\gamma_1 + \gamma_2)} \exp [-j\gamma_1 (z + h)] \quad (2.31c)$$

and

$$\tilde{\Pi}_{hlz}^0 = I_{h0} j\alpha \frac{\gamma_1 - \gamma_2}{k_1^2 (\kappa\gamma_1 + \gamma_2)} \exp [-j\gamma_1 (z + h)] \quad (2.32)$$

$$\tilde{\Pi}_{h2x} = I_{h0} \frac{1}{j\kappa(\gamma_1 + \gamma_2)} \exp (-j\gamma_1 h) \exp (j\gamma_2 z) \quad (2.33)$$

$$\tilde{\Pi}_{h2z} = I_{h0} j\alpha \frac{\gamma_1 - \gamma_2}{k_1^2 (\kappa\gamma_1 + \gamma_2)} \exp (-j\gamma_1 h) \exp (j\gamma_2 z) \quad (2.34)$$

As in the vertical case, the first two terms of the dominant vector potential component, viz., $\tilde{\Pi}_{hlx}^i$ and $\tilde{\Pi}_{hlx}^r$, can be interpreted as the direct and the perfect ground reflection contributions to the fields for observation points in region 1. Therefore, the two remaining components $\tilde{\Pi}_{hlx}^0$ and $\tilde{\Pi}_{hlz}^0$ are simply the correction terms to the perfect ground solution.

2.4 Space-Domain Representation of the Vector Potentials

Since for most practical antenna problems the observation points are located above ground, in this section, only the vector potential components

in region 1 are considered. The complete transform domain expressions for these vector potentials, both for the vertical and the horizontal current elements, are derived in the previous sections, and the inverse transform relation (2.8b) is employed to obtain the corresponding space-domain results.

By using a spherical-type change of variables, viz.,

$$\begin{cases} x = r_2 \sin \theta_2 \cos \phi_2 = \rho_2 \cos \phi_2 \\ y = r_2 \sin \theta_2 \sin \phi_2 = \rho_2 \sin \phi_2 \\ z + h = r_2 \cos \theta_2 = z_2 \end{cases} \quad (2.35)$$

$$\begin{cases} \alpha = -\lambda \cos \zeta \\ \beta = -\lambda \sin \zeta \end{cases}, \quad (2.36)$$

the incident and the perfect ground-reflection components of the vector potentials, viz., Equations (2.21a),(2.21b) and (2.31a),(2.31b) can be inverse transformed into the following general space-domain form:

$$W = \frac{I_0}{4\pi j} \int_0^\infty \frac{\lambda}{\sqrt{k_1^2 - \lambda^2}} J_0(\rho_2 \lambda) \exp[-j|z| \sqrt{k_1^2 - \lambda^2}] d\lambda \quad (2.37)$$

with the requirement that $\text{Im} \sqrt{k_1^2 - \lambda^2} \leq 0$. In deriving the preceding equation, the following identity was used

$$\cos(n\tau) J_n(z) = \frac{(-j)^{-n}}{2\pi} \int_{-\pi}^{\pi} e^{-jz \cos(\tau' - \tau)} \cos(n\tau') d\tau' \quad (2.38)$$

where J_n is the n^{th} -order Bessel function. Expression (2.37) can be integrated in a closed form [15] to yield:

$$W = I_0 \exp(-jk_1 r) / 4\pi r \quad (2.39)$$

Equation (2.39) is used to write the space-domain expressions for the aforementioned vector potential components in the following closed forms:

$$\Pi_{v1z}^i = I_{v0} \exp(-jk_1 r_1) / 4\pi r_1 \quad (2.40a)$$

$$\Pi_{v1z}^r = -I_{v0} \exp(-jk_1 r_2) / 4\pi r_2 \quad (2.40b)$$

$$\Pi_{h1x}^i = I_{h0} \exp(-jk_1 r_1) / 4\pi r_1 \quad (2.41a)$$

$$\Pi_{h1x}^r = -I_{h0} \exp(-jk_1 r_2) / 4\pi r_2 \quad (2.41b)$$

One may recognize Equations (2.40a) and (2.41a) as the free-space Green's function solution for the current element P_1 , while (2.40b) and (2.41b) are the corresponding perfect ground contributions, i.e., source at image point P_2 , to the fields at observation points in region 1.

Inverting the remaining vector potential components in region 1 and after some manipulations, one obtains:

$$0\Pi_{v1z} = \frac{2I_{v0}}{4\pi j} \int_0^\infty \frac{\lambda}{\kappa\sqrt{k_1^2 - \lambda^2} + \sqrt{\kappa k_1^2 - \lambda^2}} J_0(\rho_2 \lambda) e^{-jz_2 \sqrt{k_1^2 - \lambda^2}} d\lambda \quad (2.42)$$

$$0\Pi_{h1x} = \frac{2I_{h0}}{4\pi j} \int_0^\infty \frac{\lambda}{\sqrt{k_1^2 - \lambda^2} + \sqrt{\kappa k_1^2 - \lambda^2}} J_0(\rho_2 \lambda) e^{-jz_2 \sqrt{k_1^2 - \lambda^2}} d\lambda \quad (2.43)$$

$$0\Pi_{h1z} = -\frac{2I_{h0}}{4\pi k_1^2} \cos \phi_2 \int_0^\infty \lambda^2 \frac{\sqrt{k_1^2 - \lambda^2} - \sqrt{\kappa k_1^2 - \lambda^2}}{\kappa\sqrt{k_1^2 - \lambda^2} + \sqrt{\kappa k_1^2 - \lambda^2}} J_1(\rho_2 \lambda) e^{-jz_2 \sqrt{k_1^2 - \lambda^2}} d\lambda \quad (2.44)$$

where relations $\text{Im} \sqrt{k_1^2 - \lambda^2} \leq 0$ and $\text{Im} \sqrt{\kappa k_1^2 - \lambda^2} \leq 0$ in (2.42) - (2.44) must hold. The infinite integrals present in the above equations are

popularly known as the Sommerfeld integrals [1] and cannot be expressed in a closed form. Efficient numerical evaluation of these integrals, which have been recently verified [3] - [14], represents the major task in analyzing antenna structures over lossy ground.

The Sommerfeld integral can take several forms. For this work, the forms containing Hankel functions in their integrands are preferred, since the integrals appear to be numerically more tractable. Incorporating the well-known identities between Bessel and Hankel functions, viz.,

$$J_i(x) = \frac{1}{2} [H_i^{(1)}(x) + H_i^{(2)}(x)] \quad ; \quad i = 0, 1 \quad (2.45a)$$

$$H_0^{(1)}(x) = -H_0^{(2)}(-x) \quad (2.45b)$$

$$H_1^{(1)}(x) = H_1^{(2)}(-x) \quad , \quad (2.45c)$$

and introducing the following change of variable

$$\lambda = k_1 \sin \xi \quad , \quad (2.46)$$

the vector potential components containing the Sommerfeld integrals (2.42) - (2.44) take the following form

$$0^{\Pi}_{vlz} = \frac{I_{v0} k_1 \kappa}{4\pi j} \int \frac{\sin \xi \cos \xi}{\kappa \cos \xi + \sqrt{\kappa^2 - \sin^2 \xi}} H_0^{(2)}(k_1 \rho_2 \sin \xi) e^{-jk_1 z_2 \cos \xi} d\xi \quad (2.47)$$

$$0^{\Pi}_{h1x} = \frac{I_{h0} k_1}{4\pi j} \int \frac{\sin \xi \cos \xi}{\kappa \cos \xi + \sqrt{\kappa^2 - \sin^2 \xi}} H_0^{(2)}(k_1 \rho_2 \sin \xi) e^{-jk_1 z_2 \cos \xi} d\xi \quad (2.48)$$

and

$$\begin{aligned}
 {}_0\Pi_{hlz} = & -\frac{I_{h0}k_1}{4\pi} \cos \phi_2 \int_{\Gamma} \sin^2 \xi \cos \xi \frac{\cos \xi - \sqrt{\kappa - \sin^2 \xi}}{\kappa \cos \xi + \sqrt{\kappa - \sin^2 \xi}} \\
 & \cdot H_1^{(2)}(k_1 \rho_2 \sin \xi) e^{-jk_1 z_2 \cos \xi} d\xi
 \end{aligned} \tag{2.49}$$

where the integration path Γ is depicted in Figure 2, on which the following conditions must hold

$$\text{Im}(\cos \xi) \leq 0 \tag{2.50a}$$

$$\text{Im}(\sqrt{\kappa - \sin^2 \xi}) \leq 0 \tag{2.50b}$$

In the following chapter, an efficient numerical integration scheme is discussed for evaluating the Sommerfeld integrals via the steepest descent path integration technique. An additional approach is introduced in Chapter 4 in which the transform domain of the troublesome vector potential components are initially approximated such that the Sommerfeld infinite integrals do not appear in their space-domain expressions. These two techniques plus the asymptotic solution to the Sommerfeld integrals are compared, and several numerical results are included to demonstrate the efficiency and the accuracy of each procedure.

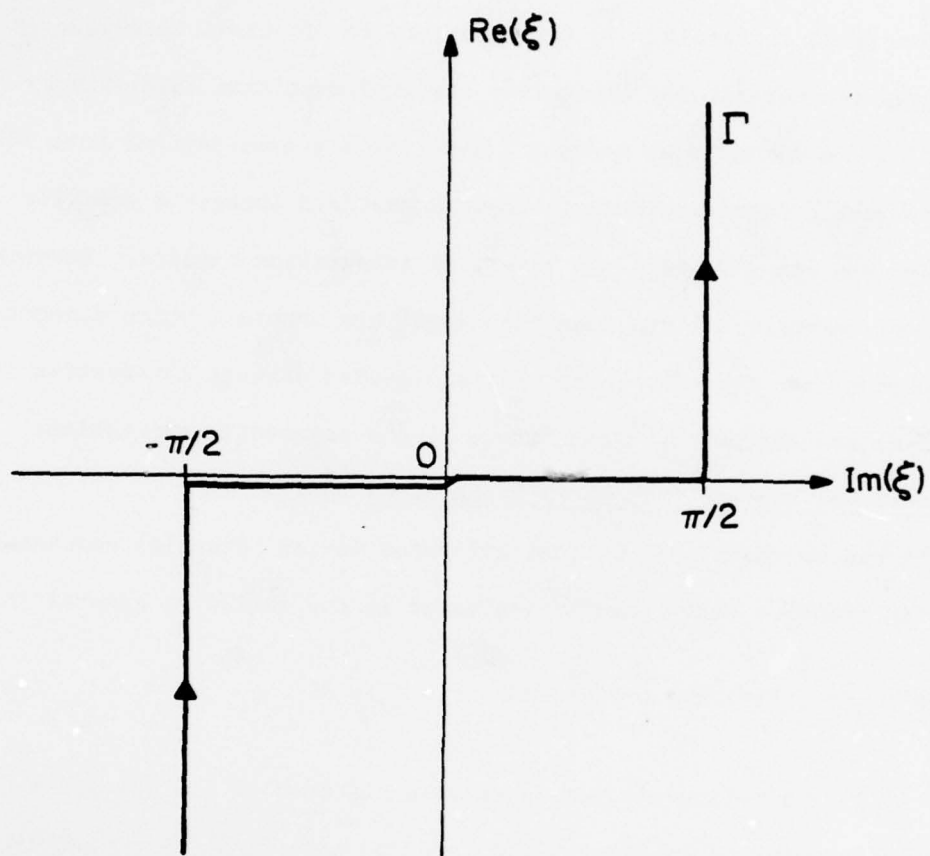


Figure 2. Integration path Γ in the complex ξ -plane.

3. EXACT AND ASYMPTOTIC EVALUATION OF THE SOMMERFELD INTEGRALS

The three correction vector potential components derived in the previous chapter, viz., Equations (2.47)-(2.49), cannot be expressed in a closed form and contain a certain class of infinite integrals known as the Sommerfeld integrals. In this chapter, an efficient technique is presented for numerically evaluating the aforementioned integrals by deforming the integration contour Γ into the steepest descent path (SDP). The asymptotic approximations to these Sommerfeld integrals are also obtained and compared with the numerical integration results. Several numerical examples are included throughout the chapter, which demonstrate the accuracy and the efficiency of the steepest descent integration technique and define the useful range of the asymptotic expressions.

3.1 Steepest Descent Path (SDP) Integration

It can be readily shown that all three vector potential components given in (2.47) - (2.49) can be expressed in the following general form:

$$u = \frac{1}{4\pi j} \int_{\Gamma} P(\xi) \exp \left[-jk_1 r_2 \cos (\xi - \theta_2) \right] d\xi \quad (3.1)$$

where $P(\xi)$ is a relatively slowly varying function of ξ . The point $\xi = \theta_2$ is a saddle point of Equation (3.1), and the path Γ can be deformed to a steepest descent path (SDP) by enforcing the condition $\text{Re}[\cos(\xi - \theta_2)] = 1$. (At this point it is assumed that no poles or branch points of $P(\xi)$ are intercepted.) On SDP, the following convenient change of variable is introduced:

$$\cos (\xi - \theta_2) = 1 - jt^2 \quad (3.2)$$

where t is a real parameter ranging from $-\infty$ to $+\infty$. Equation (3.2) can be used to explicitly define the steepest descent path as:

$$\xi_{SDP} = \pm \left[\pi/2 + j \operatorname{Ln} (t^2 + j + |t| \sqrt{t^2 + 2j}) \right] + \theta_2 \quad ; \quad t \gtrless 0, \quad (3.3)$$

or if one needs to separate the real and the imaginary behaviors of the steepest descent path, the following equivalent form can be derived

$$\xi_{SDP} = \pm \left\{ \cos^{-1} \left[\frac{-t^2 + \sqrt{t^4 + 4}}{2} \right] + j \cosh^{-1} \left[\frac{t^2 + \sqrt{t^4 + 4}}{2} \right] \right\} + \theta_2 \quad , \quad t \gtrless 0 \quad (3.4)$$

where the inverse cosine function is assumed to be between 0 and $\pi/2$

and the inverse hyperbolic function is defined as

$$\cosh^{-1} \psi = \operatorname{Ln} \left[\psi + \sqrt{\psi^2 - 1} \right] \quad . \quad (3.5)$$

The SDP can be traced in the ξ -plane, by using Equations (3.3) or (3.4) as a function of t and θ_2 (see Figure 3). Applying the change of variable in Equation (3.2) to the integral expression in Equation (3.1), we obtain

$$u = \exp(-jk_1 r_2 - j\pi/4) / (2\sqrt{2} \pi) \int_{-\infty}^{\infty} Q(t) \exp(-k_1 r_2 t^2) dt \quad (3.6)$$

where

$$Q(t) = \left[P(\xi) \sec \frac{\xi - \theta_2}{2} \right]_t = \sqrt{2} e^{j\pi/4} (t^2 + 2j)^{-1/2} P(\xi) |_t \quad . \quad (3.7)$$

Using the general form obtained in (3.6), the vector potential expressions in (2-47) - (2-49) can be formulated on the steepest descent path as a function of real variable t , namely,

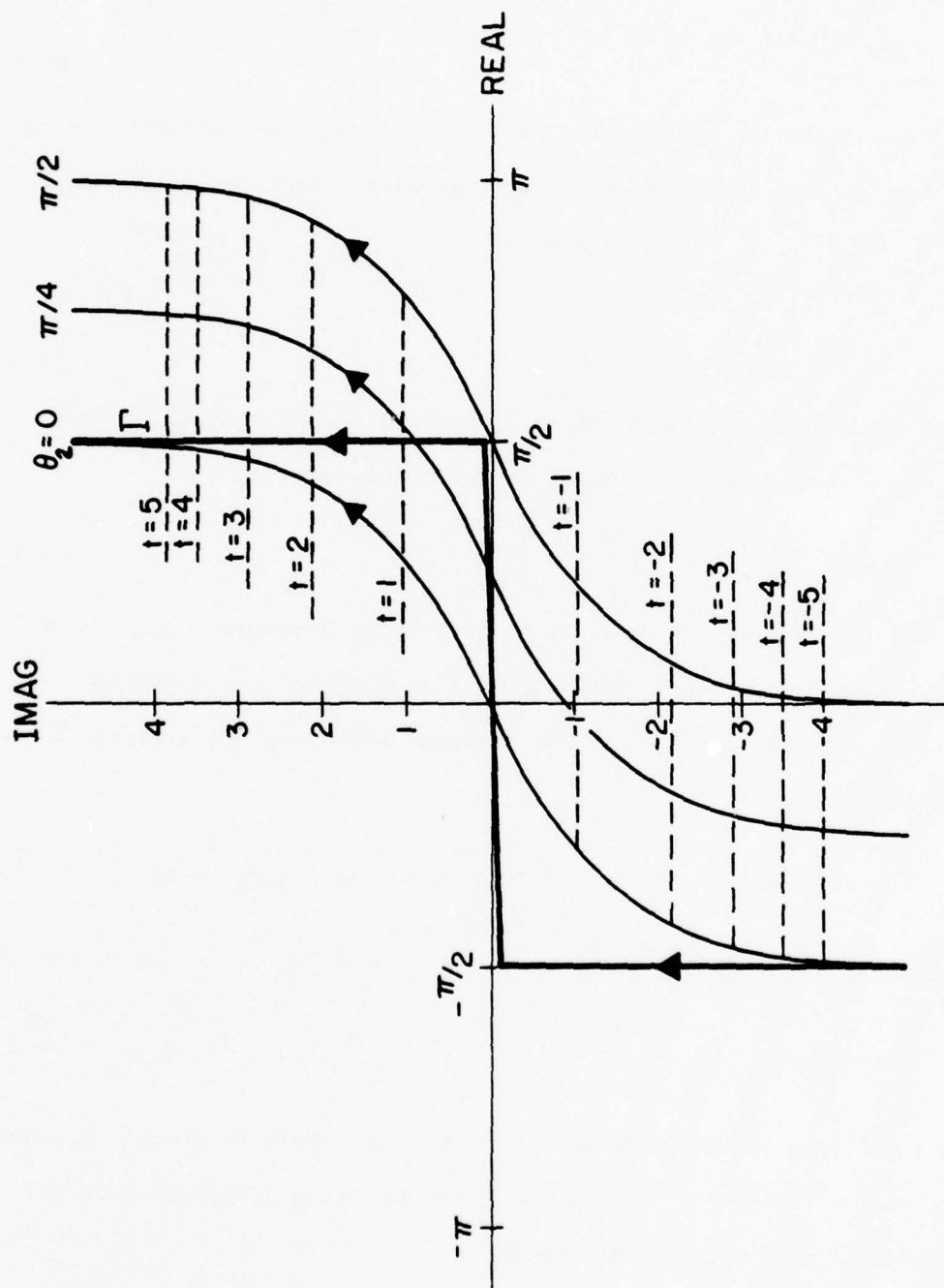


Figure 3. The steepest descent path (SDP) of integration as a function of θ_2 .

$$O_{vlz}^{\Pi} = I_{v0} \int_{-\infty}^{\infty} \frac{\kappa R_0(t)}{\left[\kappa \cos \xi + \sqrt{\kappa - \sin^2 \xi} \right]_t} \exp(-k_1 r_2 t^2) dt \quad (3.8)$$

$$O_{hlx}^{\Pi} = I_{h0} \int_{-\infty}^{\infty} \frac{R_0(t)}{\left[\cos \xi + \sqrt{\kappa - \sin^2 \xi} \right]_t} \exp(-k_1 r_2 t^2) dt \quad (3.9)$$

$$O_{hlz}^{\Pi} = -j I_{h0} \cos \phi_2 \int_{-\infty}^{\infty} R_1(t) \left[\frac{\sin \xi}{\kappa \cos \xi + \sqrt{\kappa - \sin^2 \xi}} \right]_t \cdot \exp(-k_1 r_2 t^2) dt \quad (3.10)$$

where

$$R_i(t) = (2\pi)^{-1} k_1 \exp(-jk_1 r_2) (t^2 + 2j)^{-1/2} [\sin \xi \cos \xi \exp(jk_1 \rho_2 \sin \xi)]_t \cdot H_i^{(2)}(k_1 \rho_2 \sin \xi) \Big|_t ; \quad i = 0, 1, \quad (3.11)$$

$H_i^{(2)}$ is the Hankel function of the i^{th} order and of second kind, and ξ is expressed as a function of t in Equation (3.3).

The relations expressed in Equations (3.8) - (3.10) are exact if no poles or branch points are intercepted under the path deformation. The detailed discussions of the possible pole and branch-cut contributions are presented in the following sections. It should also be noted that the apparent singularity of the Hankel functions at $\theta_2 = 0$ in the Sommerfeld integrals is overcome by the remaining terms in the integrands. In Appendix I, equivalent versions of Equations (3.8) - (3.10) are derived for $\theta_2 = 0$ for numerical integration purposes.

By observing the integral expressions in (3.8) - (3.10), it is apparent that for large $k_1 r_2$ the effective integration interval will be quite small and contain relatively few oscillations (see Figure 4).

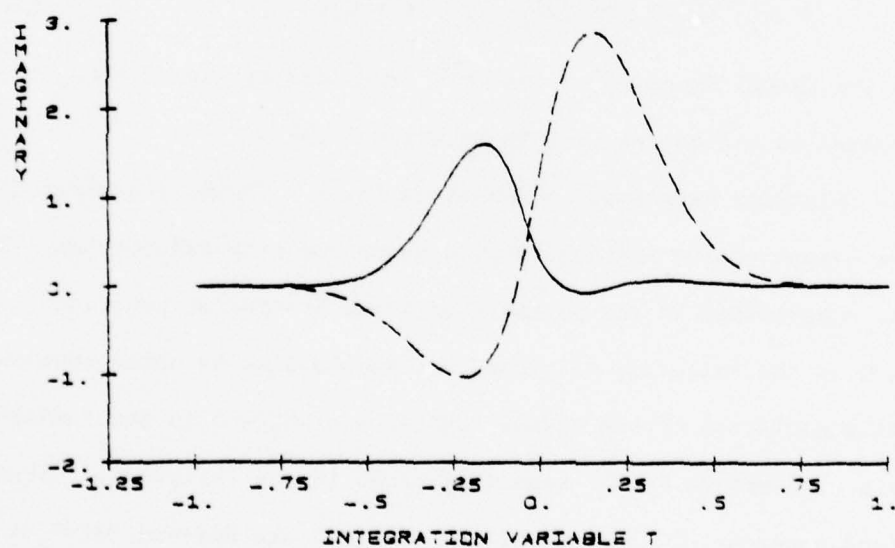
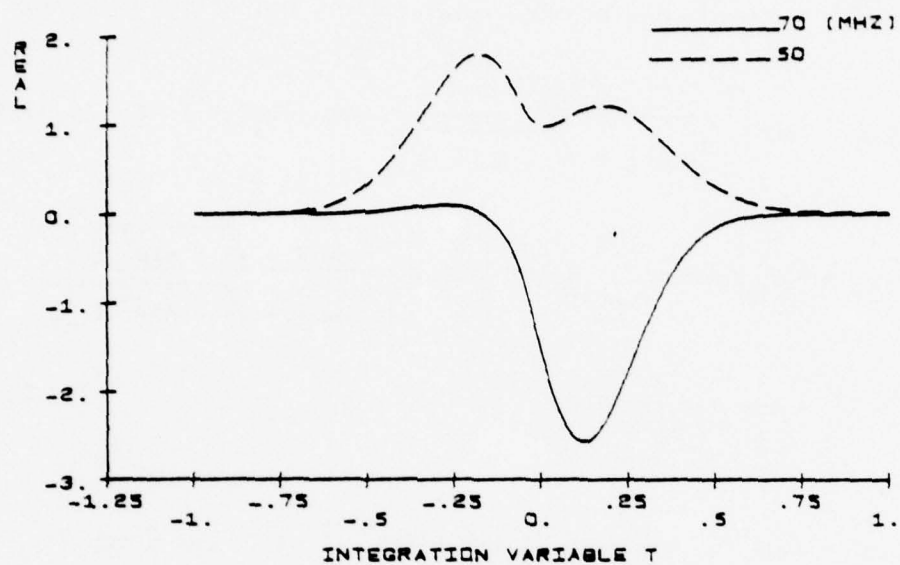


Figure 4. High-frequency examples of the Π_{h1x} . For these cases, $\theta_2 = 5^\circ$, $h = 5\text{m}$, $\epsilon_g = 10$, and $\sigma = .01$ mhos/m.

Fortunately, at lower frequencies (smaller $k_1 r_2$), the oscillatory terms present in the integrands are scaled accordingly and, even though the effective integration interval increases, the number of oscillations in the integrand does not increase appreciably (Figure 5). This fact allows one to integrate Equations (3.8) - (3.10) numerically by employing an efficient Gaussian quadrature integration routine for a wide range of parameters. Later in this chapter, the efficiency and the accuracy of the above numerical integration procedure are demonstrated and compared to the other available techniques.

3.2 Branch-Cut Contribution

The expressions derived in the previous section are valid only when no singularities are intercepted during the steepest descent path deformation. In order to locate the poles and the branch points, one must consider the following physical constraints:

- a) $0 \leq \theta_2 < \pi/2$, since Equations (3.8) - (3.10) are valid only for observation points above ground;
- b) $\text{Re}(\kappa) \geq 1$ and $\text{Im}(\kappa) \leq 0$, since $\kappa = \epsilon_g - j\sigma/(\omega\epsilon_0)$;
- c) $-\frac{\pi}{2} < \text{Re}(\xi) < \pi$ on the SDP (see Figure 3).

Furthermore, since $\cos(\xi)$ is a single-valued function, condition (2.50a) does not have to be satisfied during the path deformation. Condition (2.50b) is used to define an upper- and a lower-Riemann sheet in the ξ -plane in which this condition is satisfied in the upper and violated in the lower sheet.

Equations (3.8) - (3.10) have the same branch points satisfying

$$\kappa - \sin^2 \xi_b = 0 \quad . \quad (3.12)$$

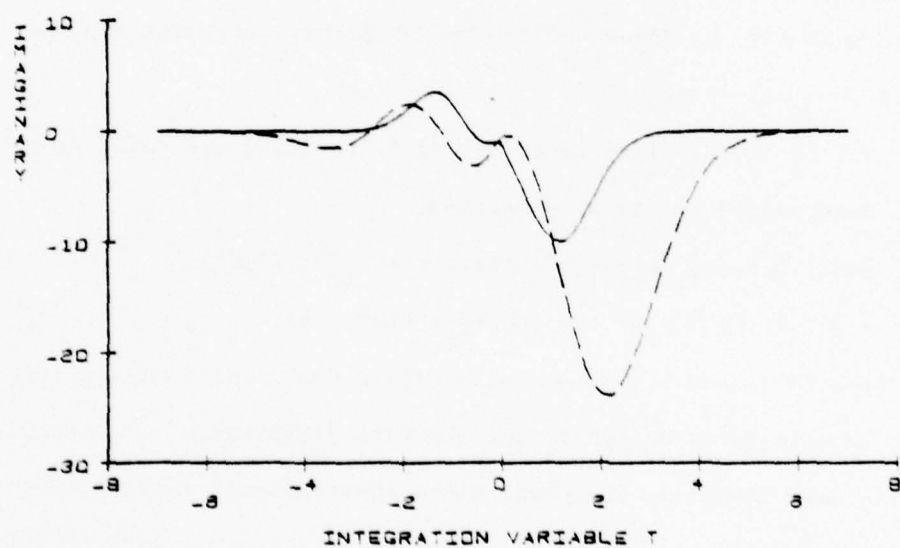
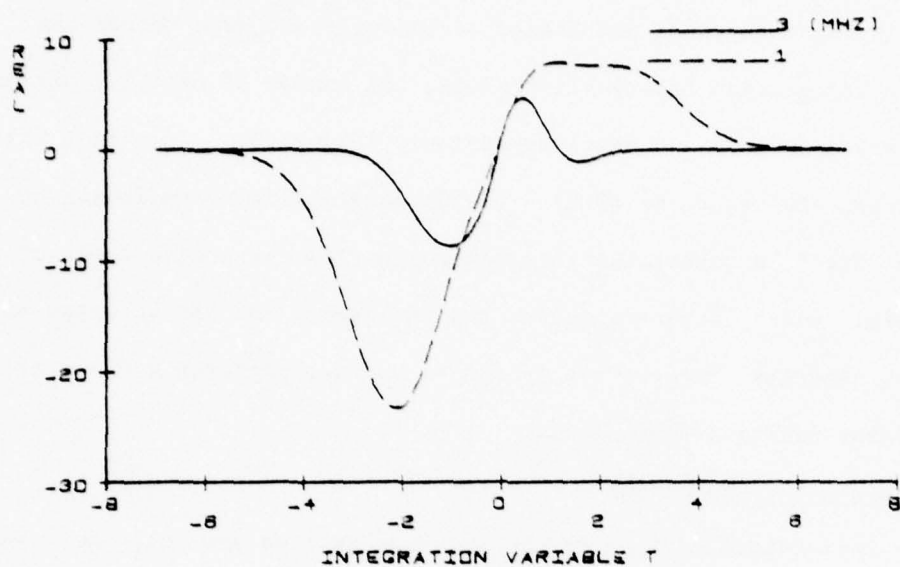


Figure 5. Low-frequency examples of the Π_{h1x} . For these cases, $\theta_2 = 5^\circ$, $h = 5\text{m}$, $\epsilon_g = 10$, and $\sigma = .01 \text{ mhos/m}$.

If one considers the physical constraints discussed earlier, only the following two branch point solutions are of importance (see Figure 6):

$$\xi_b = \pi/2 \pm j \operatorname{Ln} (\sqrt{\kappa} + \sqrt{\kappa - 1}) \quad . \quad (3.13)$$

The corresponding branch cuts of Equation (3.13) which satisfy the relation

$$\operatorname{Im} (\sqrt{\kappa - \sin^2 \xi}) = 0 \quad , \quad (3.14)$$

as depicted in Figure 6, are the boundaries through which the integration path will travel to and from the two Riemann sheets defined earlier.

Since the steepest descent path defined in Equation (3.3) is independent of κ , one can easily demonstrate that for $0 \leq \theta_2 < 90^\circ$ only the branch point with the upper sign can be captured by the SDP deformation (Figure 7).

Therefore, one can allow the SDP to enter the lower sheet only when the lower branch cut, corresponding to the lower sign of Equation (3.13), is intercepted, since the path will always intercept the lower cut at an additional point forcing it to return to the upper Riemann sheet (see Figure 7). A branch-cut integration, however, is performed around the upper branch cut whenever it is intercepted in order to remain the proper sheet. The branch cut in the upper-half plane as a function of a positive real parameter β can be expressed as

$$\xi_{bc} = \pi/2 + j \operatorname{Ln} (\sqrt{\kappa - \beta^2} + \sqrt{\kappa - 1 - \beta^2}) \quad , \quad (3.15a)$$

or, equivalently,

$$\sin \xi_{bc} = \sqrt{\kappa - \beta^2} \quad (3.15b)$$

$$\cos \xi_{bc} = -j \sqrt{\kappa - 1 - \beta^2} \quad . \quad (3.15c)$$

By applying the change of variable in Equation (3.15) to the Sommerfeld

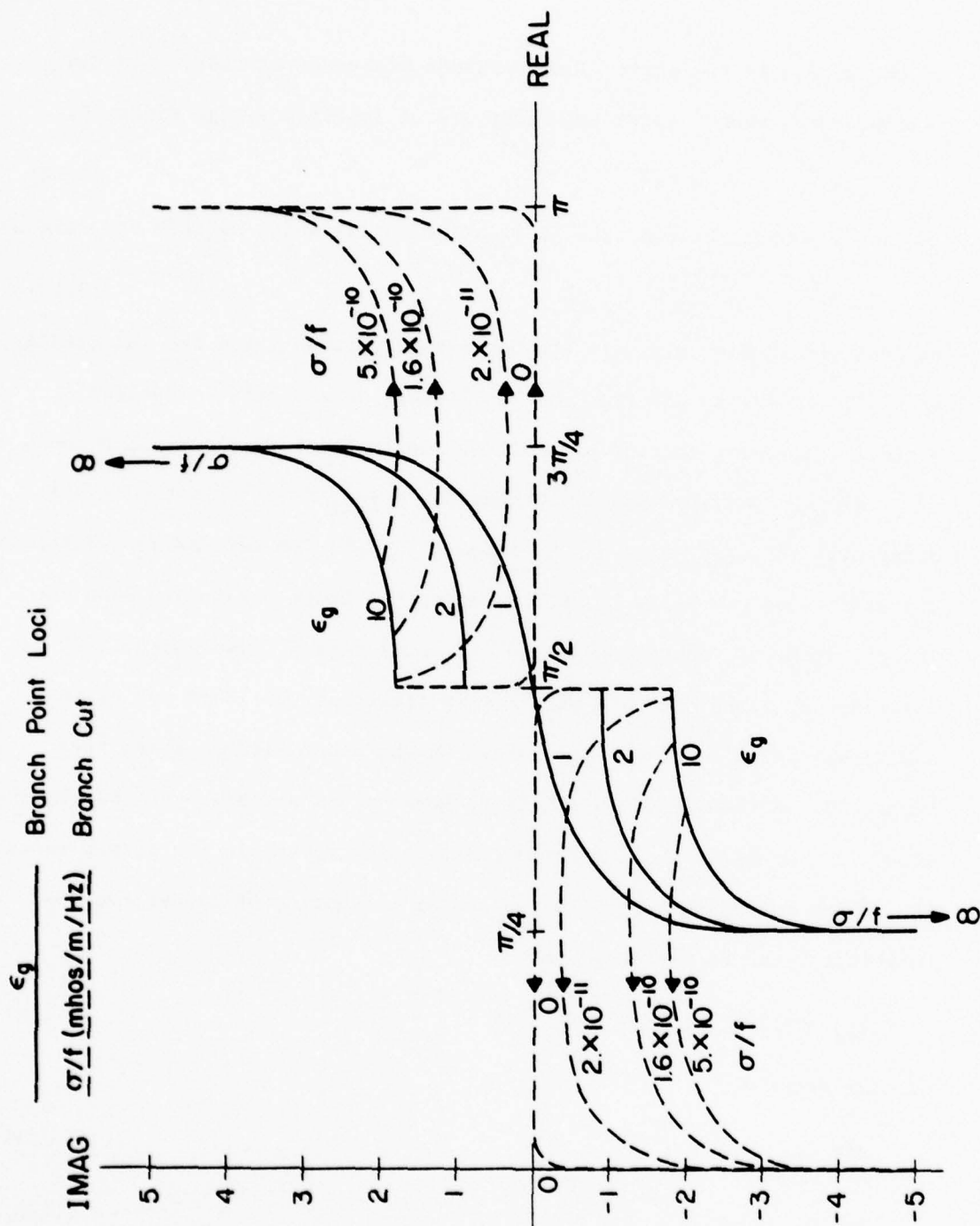


Figure 6. Branch-point and branch-cut loci as a function of ground parameters ϵ_g and σ/f .

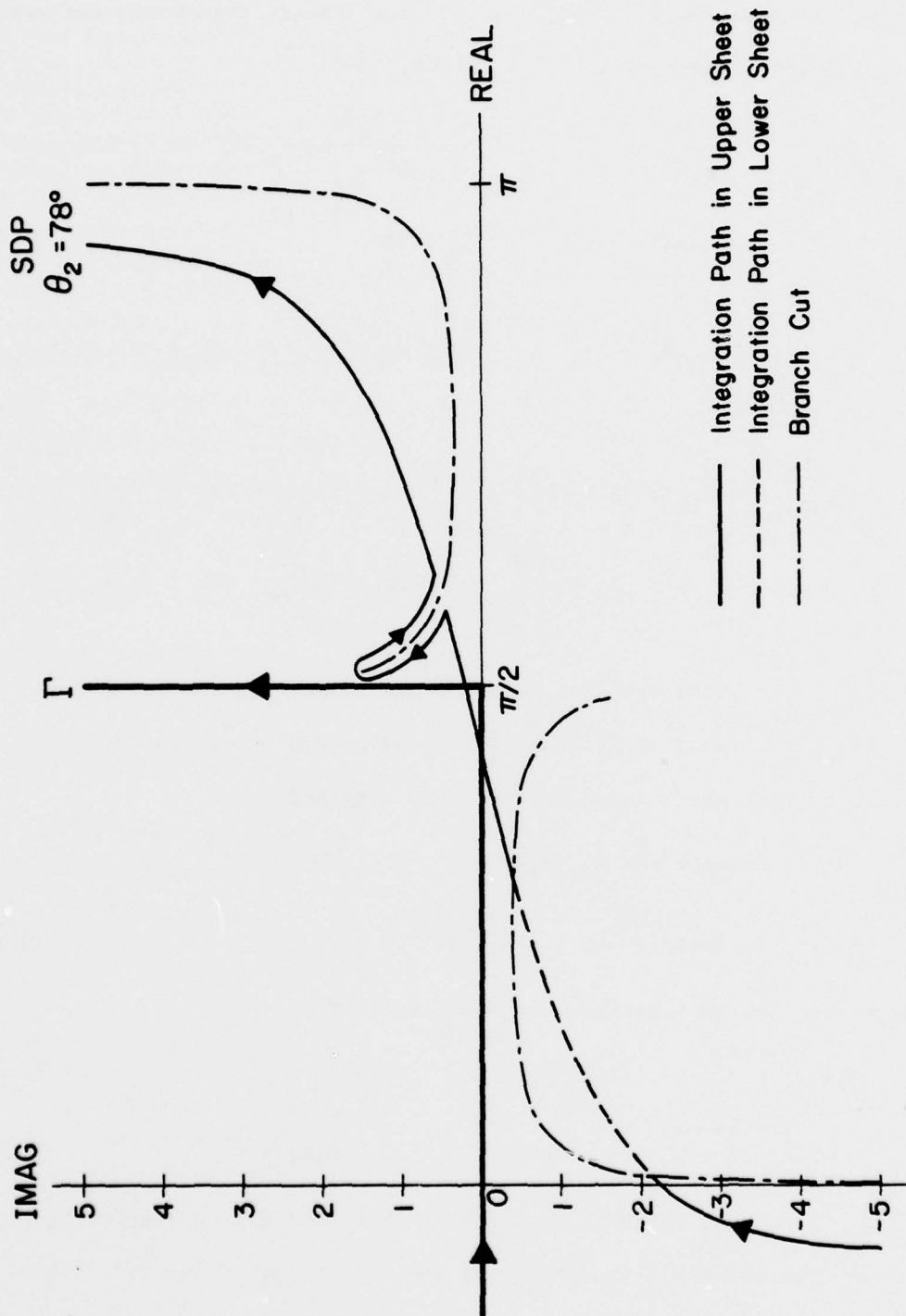


Figure 7. The correct SDP when branch cuts are intercepted. For this case, $\epsilon_g = 10$, and $\sigma/f = 2 \times 10^{-11}$ mhos/m/Hz.

integrals expressed in (2.47) - (2.49) and integrating around the branch cut, the following contributions are obtained:

$$\begin{aligned} 0^{\Pi} v_{1z} \Big|_{bc} &= I_{v0} k_1 \kappa (2\pi j)^{-1} \int_0^{\beta_1} \frac{\beta^2}{\kappa^2 \cos^2 \xi_{bc} - \beta^2} H_0^{(2)}(k_1 \rho_2 \sin \xi_{bc}) \\ &\quad \cdot \exp(-jk_1 z_2 \cos \xi_{bc}) d\beta \end{aligned} \quad (3.16)$$

$$\begin{aligned} 0^{\Pi} h_{1x} \Big|_{bc} &= I_{h0} k_1 (2\pi j)^{-1} \int_0^{\beta_1} \frac{\beta^2}{\kappa - 1} H_0^{(2)}(k_1 \rho_2 \sin \xi_{bc}) \exp(-jk_1 z_2 \cos \xi_{bc}) d\beta \\ &\quad (3.17) \end{aligned}$$

$$\begin{aligned} 0^{\Pi} h_{1z} \Big|_{bc} &= I_{h0} k_1 (2\pi)^{-1} \cos \phi_2 \int_0^{\beta_1} (1 + \kappa) \frac{\beta^2}{\kappa^2 \cos^2 \xi_{bc} - \beta^2} \\ &\quad \cdot \sin \xi_{bc} \cos \xi_{bc} H_1^{(2)}(k_1 \rho_2 \sin \xi_{bc}) \exp(-jk_1 z_2 \cos \xi_{bc}) d\beta, \\ &\quad (3.18) \end{aligned}$$

where $\beta = \beta_1$ is the crossing point of the branch cut and the SDP in the ξ -plane. By expanding Equation (3.2) and by employing the (3.15) relations, the following conditions are obtained for β_1

$$\sin \theta_2 \operatorname{Re}(A) + \cos \theta_2 \operatorname{Im}(B) = 1 \quad (3.19a)$$

$$[-\sin \theta_2 \operatorname{Im}(A) + \cos \theta_2 \operatorname{Re}(B)]^{1/2} = t_1 \quad (3.19b)$$

where the complex numbers A and B are defined as

$$A = \sqrt{\kappa - \beta_1^2} \quad ; \quad \operatorname{Re}(A) \geq 0 \quad , \quad \operatorname{Im}(A) \leq 0 \quad (3.20a)$$

$$B = \sqrt{\kappa - 1 - \beta_1^2} \quad ; \quad \operatorname{Re}(B) \geq 0 \quad , \quad \operatorname{Im}(B) \leq 0 \quad (3.20b)$$

In (3.19), $t = t_1$ defines the point at which the SDP, corresponding to the observation angle θ_2 , intercepts the branch cut of Equation (3.15) at $\beta = \beta_1$. After some algebraic manipulations, Equation (3.19a) can be

further simplified to

$$\theta_2 = \sin^{-1} ([\operatorname{Re}^2(A) + \operatorname{Im}^2(B)]^{-1/2}) - \tan^{-1} \left(\frac{\operatorname{Im}(B)}{\operatorname{Re}(A)} \right) \quad (3.21)$$

from which θ_{\min} , the observation angle at which SDP will pass through the branch point, can be computed by setting $\beta_1 = 0$. Since θ_{\min} is only a function of κ , Figure 8 is constructed to show its variations as a function of the ground parameters and the frequency. Therefore, whenever the observation angle θ_2 satisfies the condition

$$\theta_2 > \theta_{\min} \quad , \quad (3.22)$$

the branch-cut contributions in (3.16) - (3.18) are to be added to their respective SDP vector potential formulations expressed in (3.8) - (3.10).

It should be pointed out that once the condition (3.22) is met, the branch-cut integration limit β_1 can be computed numerically by iterating on Equation (3.21). Also, because of the branch-cut interception, the SDP integrand will be discontinuous at point $t = t_1$, which is readily computed by substituting the value of β_1 into (3.19b).

Fortunately, in many cases, the branch-cut contributions are several orders of magnitude smaller than the SDP integral value and can be ignored [16]. Therefore, it is necessary to introduce a condition for which one can ignore the branch-cut integration and thereby compute the vector potentials more efficiently. This task can be accomplished by initially considering the $\exp(-k_1 r_2 t^2)$ term present in all three vector potential integrands shown in (3.8) - (3.10). If no poles are present on or near the contour, a finite integration in the interval

$$|t| \leq \left(\frac{9}{k_1 r_2} \right)^{1/2} \quad (3.23)$$

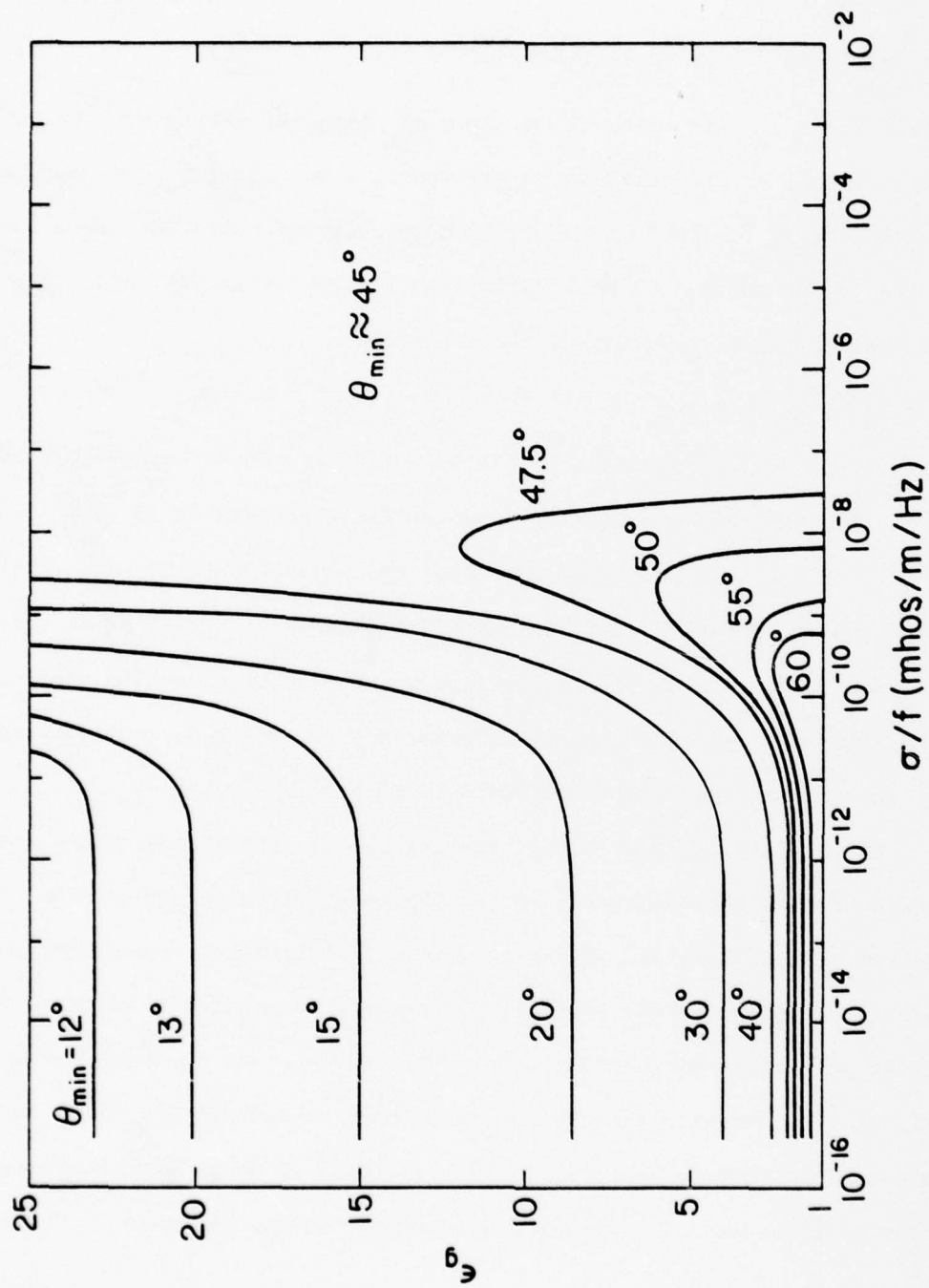


Figure 8. The minimum θ_2 contours as a function of κ . A branch point is captured by the SDP path. deformation when $\theta_2 > \theta_{\min}$.

will result in an error on the order of 0.01% as compared to the full infinite integration. By examining the branch-cut loci (Figure 6) and the SDP behavior (Figure 3) in the ξ -plane, an assumption can be made which states that if the branch cut intercepts the SDP inside the finite integration interval defined in (3.23), then the branch-cut contribution is not negligible. Or equivalently, the branch-cut contribution requires an additional condition, namely,

$$t_1 \leq t_{\max} = 3(k_1 r_2)^{-1/2} \quad (3.24)$$

where $t = t_1$ is the SDP and branch-cut intercept defined earlier.

In summary, condition (3.22) signals the capture of the branch point during the path deformation. If captured, condition (3.24) is used to decide whether the branch-cut integration can be ignored or not. Table 3.1 is constructed to verify the validity of the assumptions which led to Equation (3.24) by comparing the branch-cut values with the SDP integration results for a wide range of parameters.

3.3 Pole Contribution

Unlike the branch points, which exist in all three of the correction vector potential components, the poles only exist in the vertical components of the vector potentials, viz., 0^{Π}_{hlz} and 0^{Π}_{vlz} , and satisfy the relation

$$\kappa \cos \xi_p + \sqrt{\kappa - \sin^2 \xi_p} = 0 \quad (3.25)$$

Again, by considering the physical constraints discussed at the start of the previous section, only the following two poles need to be considered (see Figure 9):

$$\xi_p = \pi/2 \pm j [\ln (\sqrt{\kappa} - j) - \ln (\sqrt{\kappa} + 1)] \quad (3.26)$$

Table 3.1 Demonstrating the branch-cut contribution for Π_{0v12} . In this example: $f = 30$ MHz, $\phi_2 = 0$, $\epsilon_g = 10$, $\sigma = .001$ mhos/m, and $I_{v0} = 1$.

r_2 (meters)	θ_2	t_{\max}	t_1	β_1	SDP integration result $\times 10^4$	Branch-cut contribution $\times 10^4$	Total value $\times 10^4$
20.	0.	.85	---	---	55.2-j2.07	---	55.2-j2.07
20.	30.	.85	---	---	53.2-j2.47	---	53.2-j2.46
20.	60.	.85	.81	1.8	44.6-j4.74	$-7.3 \times 10^{-4} + j1.3 \times 10^{-3}$	44.6-j4.74
20.	90.	.85	.55	2.0	14.5-j20.9	$-.223 + j.308$	14.3-j20.6
10.	0.	1.2	---	---	111.6-j6.02	---	111.1-j6.02
10.	30.	1.2	---	---	107.6-j7.38	---	107.6-j7.38
10.	60.	1.2	.81	1.8	93.0-j14.4	$-.128 + j.204$	92.9-j14.2
10.	90.	1.2	.55	2.0	50.2-j47.1	$-3.85 + j2.45$	46.3-j44.7
5.	0.	1.7	---	---	-226.5+j17.5	---	-226.5+j17.5
5.	30.	1.7	---	---	-221.3+j21.3	---	-221.3+j21.3
5.	60.	1.7	.81	1.8	-199.1+j36.0	3.17-j3.61	-195.9+j32.4
5.	90.	1.7	.55	2.0	-140.3+j86.6	16.5-j22.7	-123.8+j63.9

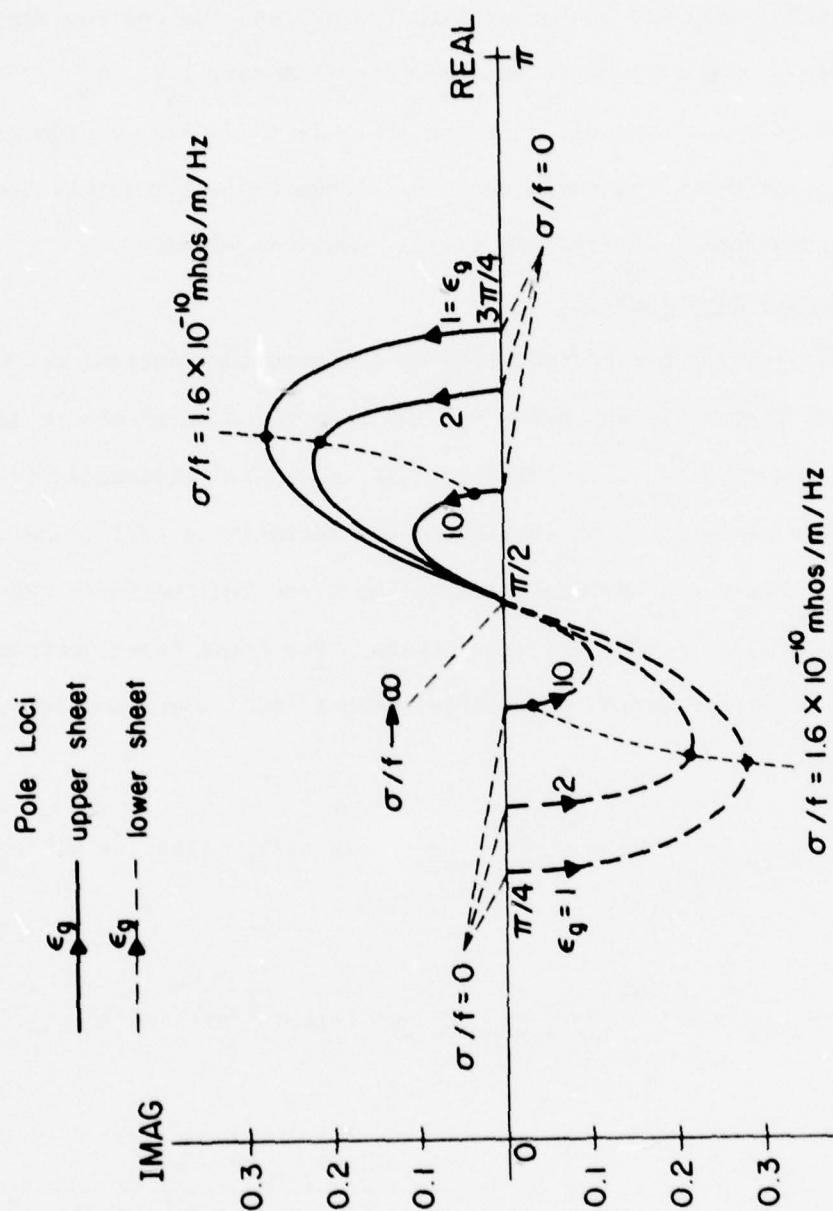


Figure 9. Pole loci as a function of ground parameters ϵ_g and σ/f .

Fortunately, one can verify that for $0 \leq \sigma/f < \infty$, $0 \leq \theta_2 < 90^\circ$, and by considering the correct Riemann sheets (see Figure 7), the poles in Equation (3.26) will not be captured by the SDP and the residue contribution is not needed. However, under extreme circumstances, i.e., $\theta_2 \approx 90^\circ$ and small ϵ_g , a pole can come close to the SDP. In this case, a higher-order Gaussian quadrature integration routine is required and possibly the effective integration interval in (3.23) should be expanded.

3.4 Asymptotic Approximation

The steepest descent formulation of the previous sections can be used to derive an asymptotic expansion for the Sommerfeld integrals in terms of inverse powers of $k_1 r_2$. In Appendix II, a general discussion is presented for asymptotically evaluating the integral in (3.1), and as an example, the first two asymptotic expansion terms for the vector potential expressions in (2.47) - (2.49) are derived. The first terms, better known as the Fresnel's reflection coefficient method (RCM) approximations, are shown to be:

$$0^{\text{II}}_{\text{v}1z} = I_{\text{v}0} \frac{2\kappa \cos \theta_2}{\kappa \cos \theta_2 + \sqrt{\kappa^2 - \sin^2 \theta_2}} \exp(-jk_1 r_2)/4\pi r_2 + O(k_1 r_2)^{-2} \quad (3.27)$$

$$0^{\text{II}}_{\text{h}1x} = I_{\text{h}0} \frac{2 \cos \theta_2}{\cos \theta_2 + \sqrt{\kappa^2 - \sin^2 \theta_2}} \exp(-jk_1 r_2)/4\pi r_2 + O(k_1 r_2)^{-2} \quad (3.28)$$

$$0^{\text{II}}_{\text{h}1z} = I_{\text{h}0} 2 \cos \phi_2 \sin \theta_2 \cos \theta_2 \frac{\cos \theta_2 - \sqrt{\kappa^2 - \sin^2 \theta_2}}{\kappa \cos \theta_2 + \sqrt{\kappa^2 - \sin^2 \theta_2}} \cdot \exp(-jk_1 r_2)/4\pi r_2 + O(k_1 r_2)^{-2} \quad (3.29)$$

The above RCM expressions have been extensively used for high-frequency antenna applications ($k_1 r_2$ large) with surprisingly accurate results, e.g., [13], [14]. This success has been mainly due to the fact that for observation points away from the interface, the remaining vector potential components present in (2.20) and (2.30) tend to dominate over the RCM components thereby reducing the net error in the total vector potential values computed. Therefore, little is gained by adding the complicated second terms in the asymptotic expansion of the vector potentials, derived in Appendix II, since the accuracy of the total vector potential will not be affected appreciably.

An error of 5% or less can be expected in the RCM vector potential components shown in (3.27) - (3.29) when

$$k_1 r_2 \geq 10 \quad , \quad (3.30)$$

and as long as the branch-cut conditions given in the previous section are not violated (see Tables 3.2 - 3.4). These conditions are more general than the one proposed by Sarkar [7] in defining the useful range of the RCM expressions. The second terms in the asymptotic expansion, as demonstrated in Tables 3.2 - 3.4, only offer a slight improvement in the accuracy of the $\prod_{0 \text{ hlx}}$ RCM expression in the region where the branch-cut contribution is negligible. The remaining vector potential components, however, do not benefit from the 2nd term in the asymptotic expansions, possibly because they contain a pole in their Sommerfeld integrals. Figures 10 - 12 also compare the RCM, two-term asymptotic expansion and the exact integration values of the correction vector potential components for a typical half-space as a function of $k_1 r_2$. Finally, in order to

Table 3.2 Comparing the exact integration values for 0^{II}_{v12} with its one- and two-term asymptotic expansions.
 In this example, $f = 30$ MHz, $\phi_2 = 0$, $\epsilon_g = 10$, $\sigma = .001$ mhos/m, and $I_{v0} = 1$.

$k_1 r_2$	θ_2	1-term asymptotic expansion (RCM) $\times 10^3$	2-term asymptotic expansion $\times 10^3$	SDP integration result $\times 10^3$
1.	20.	35.7-j58.0	57.3-j51.8	34.6-j68.2
4.	20.	-10.9+j13.1	-12.2+j12.5	-10.4+j14.1
8.	20.	-1.40-j8.40	-1.08-j8.55	-1.69-j8.41
12.	20.	4.85+j2.96	4.80+j3.11	4.93+j2.85
1.	60.	28.6-j46.8	64.4-j55.2	29.1-j59.0
4.	60.	-8.73+j10.6	-11.0+j10.8	-7.97+j1.31
8.	60.	-1.15-j6.76	-.813-j7.23	-2.02-j6.96
12.	60.	3.91+j2.37	3.97+j2.62	4.23+j2.08
1.	80.	15.5-j26.0	-192.9-j169.2	19.8-j48.6
4.	80.	-4.76+j5.87	6.87+j16.6	-4.54+j11.5
8.	80.	-.668-j3.72	-4.59-j3.27	-2.82-j4.69
12.	80.	2.17+j1.28	3.16-j.168	3.24+j.619

Table 3.3 Comparing the exact integration values for $0^{II}x$ with its one- and two-term asymptotic expansions.
 In this example, $f = 30$ MHz, $\phi_2 = 0$, $\epsilon_g = 10$, $\sigma = .001$ mhos/m, and $I_{h0} = 1$.

$k_1 r_2$	θ_2	1-term asymptotic expansion (RCM) $\times 10^3$	2-term asymptotic expansion $\times 10^3$	SDP integration result $\times 10^3$
1.	20.	17.1-j24.3	17.2-j34.3	14.1-j36.0
4.	20.	-5.10+j5.42	-4.24+j6.17	-4.48+j6.32
8.	20.	-.383-j3.70	-.667-j3.66	-.665-j3.70
12.	20.	2.03+j1.42	2.10+j1.31	2.11+j1.32
1.	60.	11.4-j15.8	-10.0-j31.7	7.76-j36.9
4.	60.	-3.38+j3.50	-2.19+j4.67	-2.26+j4.84
8.	60.	-.217-j2.42	-.634-j2.40	-.635-j2.41
12.	60.	1.32+j.946	1.43+j.796	1.43+j.799
1.	80.	4.72-j6.36	-17.0-j24.2	.329-j39.8
4.	80.	-1.39+j1.41	-.208+j2.71	.405+j2.31
8.	80.	-.077-j.987	-.516-j.975	-.537-j.953
12.	80.	.532+j.391	.655+j.240	.651+j.236

Table 3.4 Comparing the exact integration values for 0_{hlz}^{II} with its one- and two-term asymptotic expansions.
 In this example, $f = 30$ MHz, $\phi_2 = 0$, $\epsilon_g = 10$, $\sigma = .001$ mhos/m, and $I_{h0} = 1$.

$k_1 r_2$	θ_2	1-term asymptotic expansion (RCM) $\times 10^3$	2-term asymptotic expansion $\times 10^3$	SDP integration result $\times 10^3$
1.	20.	-3.18+j5.00	-7.88+j3.04	-3.84+j6.20
4.	20.	.963-j1.13	1.24-j.963	.895-j1.33
8.	20.	.112+j.733	.036+j.758	.172+j.739
12.	20.	-.418-j.263	-.405-j.296	-.437-j.242
1.	60.	-8.08+j12.5	-17.7+j16.5	-12.9+j21.9
4.	60.	2.44-j2.80	3.07-j2.97	1.97-j4.50
8.	60.	.261+j1.84	.190+j1.98	.739+j1.93
12.	60.	3.91+j2.37	3.97+j2.62	4.23+j2.08
1.	80.	-5.96+j9.14	84.9+j79.0	-20.6+j34.1
4.	80.	1.80-j2.05	-3.21-j7.17	.419-j6.80
8.	80.	.189+j1.35	1.98+j1.24	1.48+j1.84
12.	80.	-.764-j.493	-1.25+j.140	-1.32-j.111

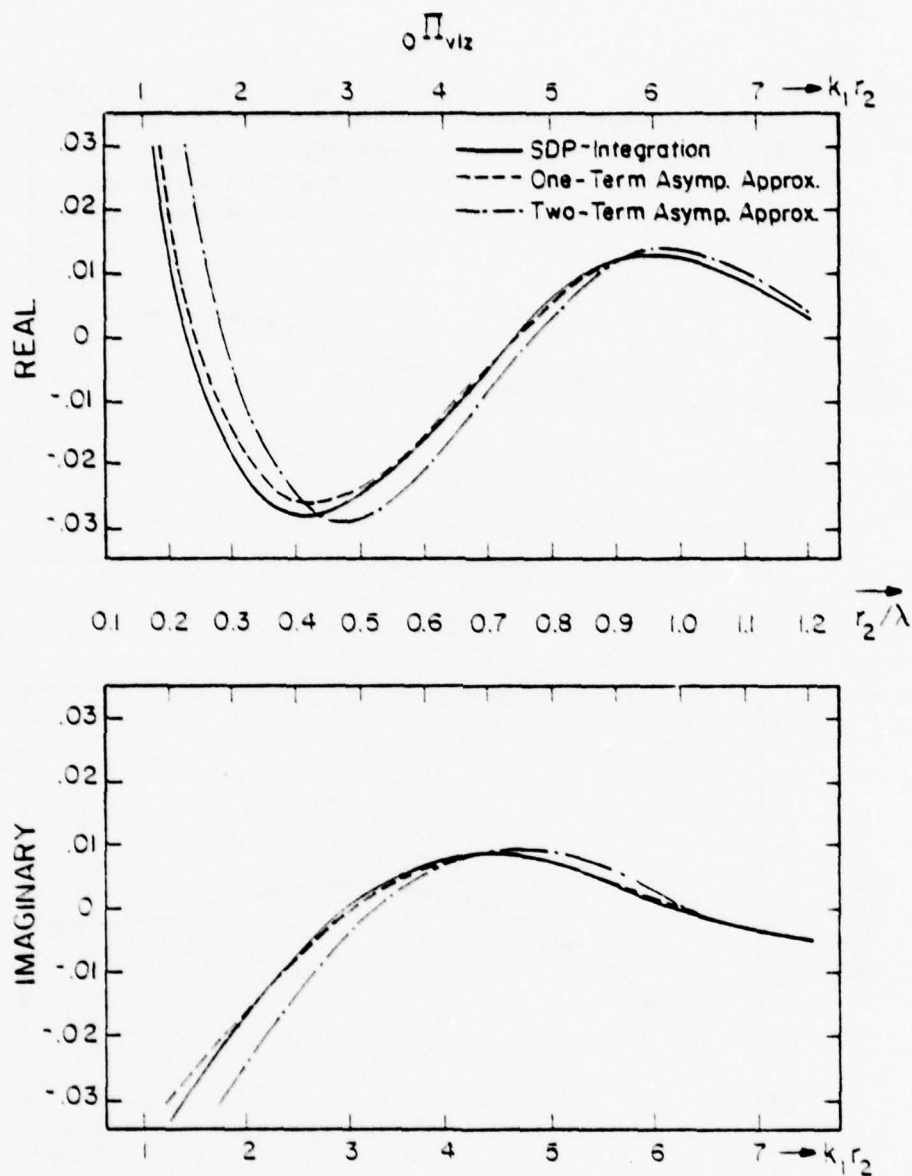


Figure 10. Comparing the SDP integration with the one- and the two-term asymptotic expansions of $0 \Pi_{v12}$. For this case, frequency = 30 MHz, $\theta_2 = 10^\circ$, $\phi_2 = 0$, $\epsilon_g = 10^0$, and $\sigma = .01$ mhos/m.

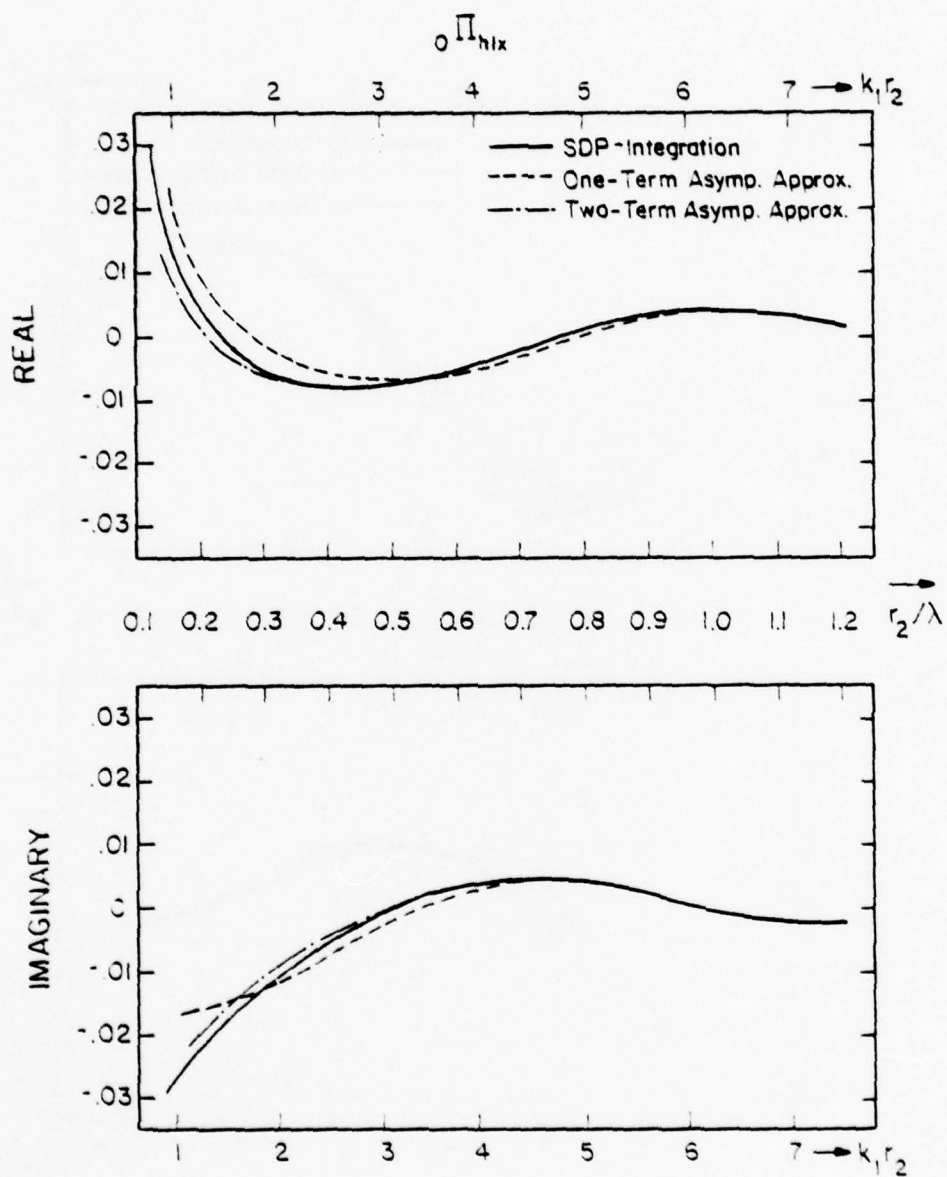


Figure 11. Comparing the SDP integration with the one- and two-term asymptotic expansions of ${}_0\Pi_{hlx}$. The parameters are identical to those in Figure 10.

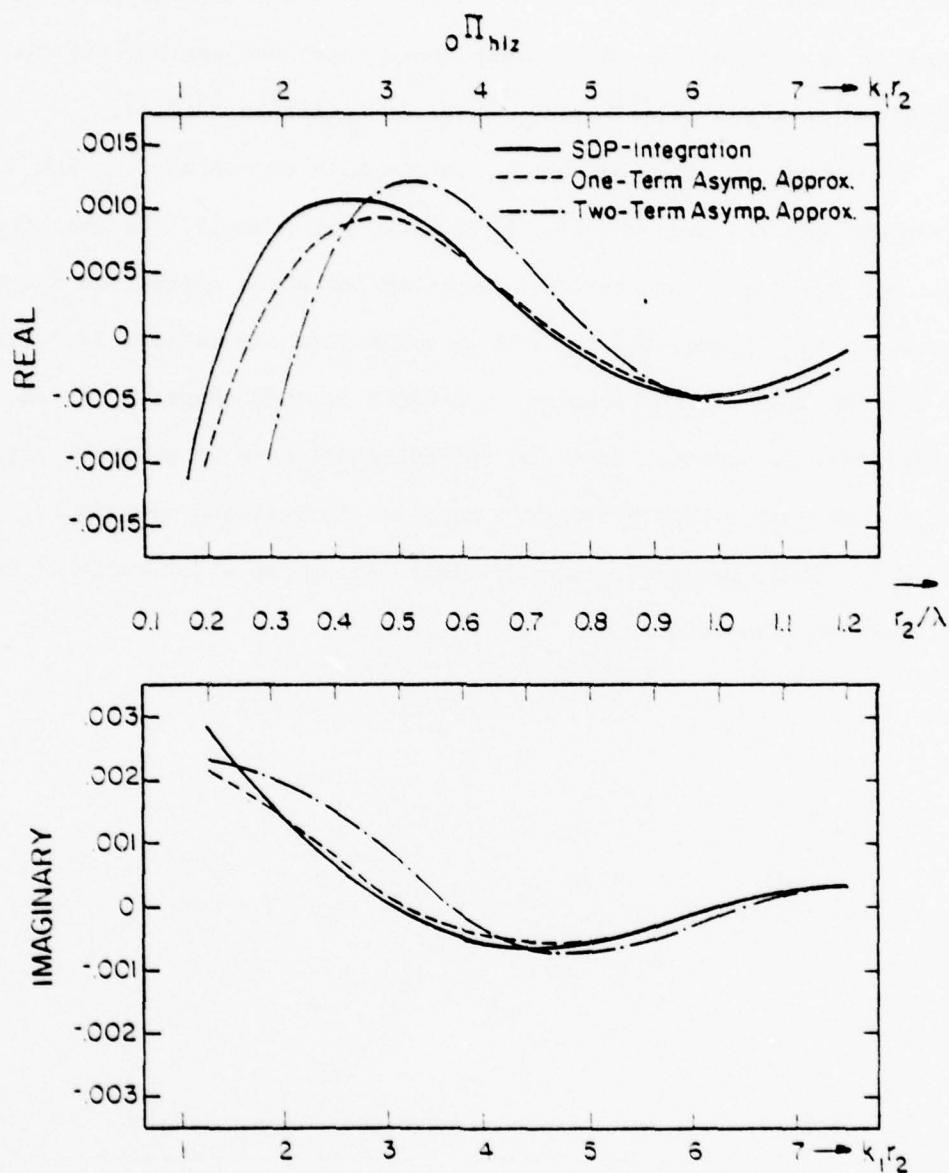


Figure 12. Comparing the SDP integration with the one- and the two-term asymptotic expansions of $o \Pi_{hlz}$. The parameters are identical to those in Figure 10.

demonstrate the branch-cut effect, Figure 13 is included to show the minimum $k_1 r_2$ contours in the κ -plane above which one can neglect the possible branch-cut contribution (based on condition (3.24)).

The computing time, of course, is the main reason why the RCM approximations are used whenever possible. For example, the execution time on a Cyber 175 computer for computing the three correction vector potentials is ≈ 1 msec for the RCM approximation and as high as 50-100 msec for the SDP integration technique presented in this chapter. It should be pointed out, however, that the SDP integration is an order of magnitude faster than many of the previously reported integration techniques [3] - [9], [12], making it more suitable for a much wider range of low-frequency applications.

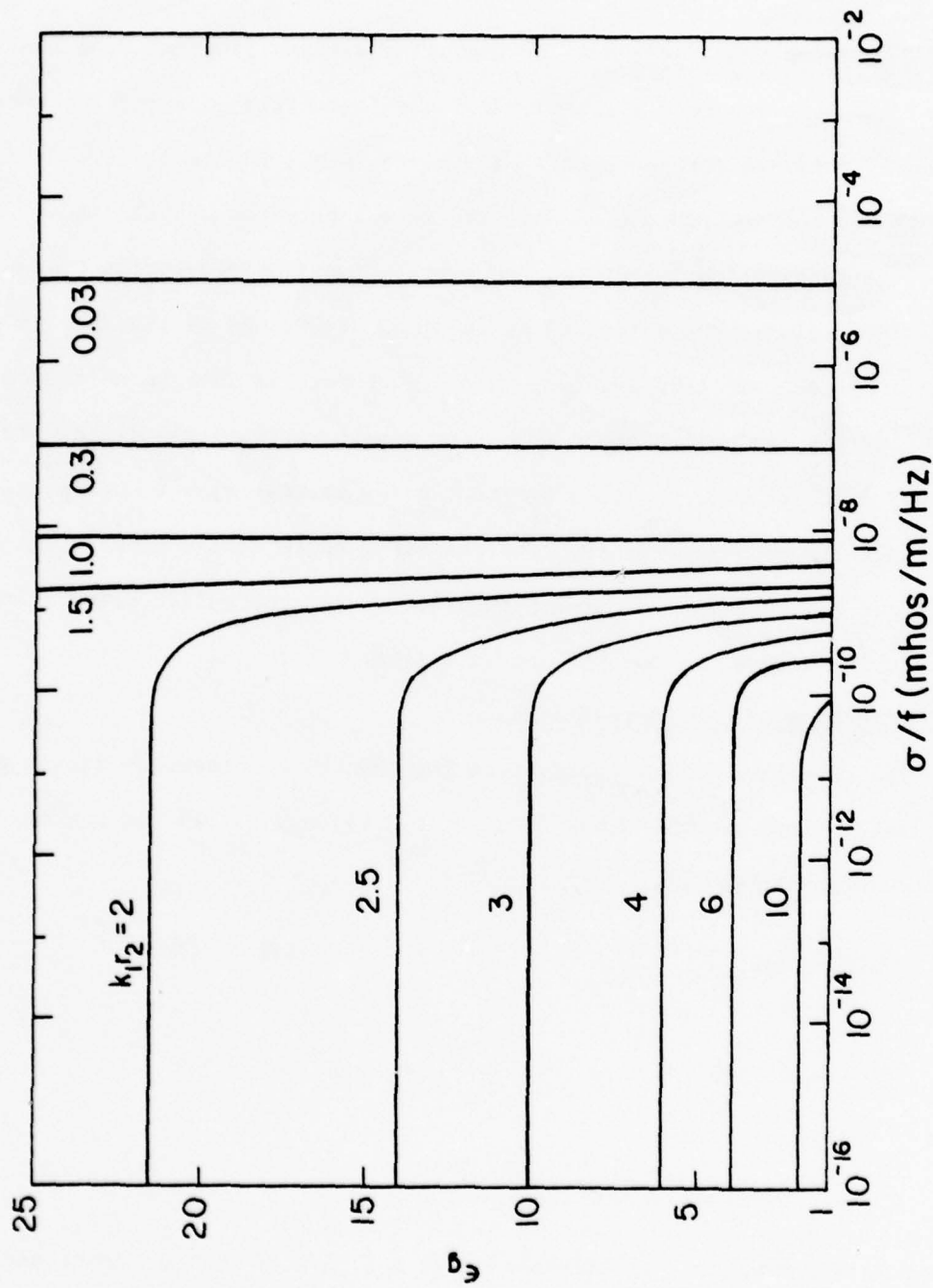


Figure 13. Minimum $k_1 r_2$ contours as a function of σ . Once a branch point is captured, a branch-cut integration is negligible if the $k_1 r_2$ is greater than the minimum values depicted in this graph.

4. COMPUTATIONS OF THE VECTOR POTENTIALS WITHOUT SOMMERFELD INTEGRATION

The conventional approach for analyzing antenna problems over lossy ground involves repeated evaluations of the Sommerfeld integrals appearing in the expressions for the vector potentials, viz., Equations (3.8) - (3.10). Even though the SDP procedure discussed in the previous chapter is an efficient integration technique, the computation time needed for evaluating these integrals severely limits the physical dimensions of the antenna problems being analyzed. In this chapter, a novel approach is introduced by initially approximating the well-behaved transform domain expressions of these integrals, derived in Chapter 2, and then performing the inverse transform operations via a set of exact identities. The resulting space domain expressions are valid for a wide range of parameters, with their computation times being comparable to those of the RCM approximation.

4.1 Transform Domain Expressions

The transform domain expressions for the three correction vector potential components shown in Equations (2.21c), (2.31c), and (2.32) can be rewritten in the following forms:

$$\tilde{0}^{\Pi}_{h1x} = I_{h0} \frac{1}{jk_1^2(1 - \kappa)} (\gamma_1 - \gamma_2) \exp(-j\gamma_1 z_2) \quad (4.1)$$

$$\tilde{0}^{\Pi}_{h1z} = I_{h0} \frac{j\alpha}{k_1^2 \kappa} \left[1 - (\kappa + 1) \frac{\gamma_2}{\kappa\gamma_1 + \gamma_2} \right] \exp(-j\gamma_1 z_2) \quad (4.2)$$

$$\tilde{0}^{\Pi}_{v1z} = I_{v0} \frac{\kappa}{j(\kappa\gamma_1 + \gamma_2)} \exp(-j\gamma_1 z_2) \quad (4.3)$$

where $z_2 = z + h$, and $\gamma_{1,2}$, defined in (2.15), are explicitly expressed as

$$\gamma_1 = (k_1^2 - \alpha^2 - \beta^2)^{1/2} \quad ; \quad \text{Im}(\gamma_1) \leq 0 \quad (4.4a)$$

$$\gamma_2 = (\kappa k_1^2 - \alpha^2 - \beta^2)^{1/2} \quad ; \quad \text{Im}(\gamma_2) \leq 0 \quad (4.4b)$$

In addition, the free-space Green's function g can also be expressed in the transform domain (see Equations (2.41b) and (2.31b)) yielding the following transform pair:

$$\tilde{g}(\alpha, \beta, z_2) = \frac{1}{2j\gamma_1} \exp(-j\gamma_1 z_2) \quad (4.5a)$$

$$g(x, y, z_2) = \exp(-jk_1 r_2)/4\pi r_2 \quad ; \quad r_2 = [x^2 + y^2 + z_2^2]^{1/2} \quad (4.5b)$$

By applying a successive $\partial/\partial z$ operator to (4.5), an infinite set of transform pairs is obtained, viz.,

$$\tilde{Q}_n = \gamma_1^{n-1} \exp(-j\gamma_1 z_2) \quad (4.6a)$$

$$Q_n = 2(j)^{n+1} \frac{\partial^n}{\partial z^n} g(x, y, z_2) \quad ; \quad n = 0, 1, 2, \dots \quad (4.6b)$$

where Q is expressed in a closed form for all n and can be numerically evaluated quite rapidly (see Appendix III).

An examination of the Fourier transformed vector potentials in (4.1) - (4.3) and the \tilde{Q}_n in (4.6a) reveals two important and useful properties. First, all of the expressions have an identical z -variation term that corresponds to a space-domain solution emanating at the image point P_2 . Second, it is apparent that all of the equations are well-behaved in the Fourier domain and decay exponentially to zero outside the circle $\alpha^2 + \beta^2 = k_1^2$. These properties give rise to the possibility of performing the inverse transform operation on Equations (4.1) - (4.3) via the use of the (4.6) identities. The major obstacle to such a procedure is the existence

of γ_2 in these expressions which is overcome in the following section by an appropriate approximation.

4.2 Approximating γ_2 and Space-Domain Results

All three Fourier transform domain expressions in (4.1) - (4.3) can be put into the following general form

$$\tilde{\Pi}_0 = f(\gamma_1, \gamma_2) \exp(-j\gamma_1 z_2) \quad (4.7)$$

As mentioned in the previous section, an important property of Equation (4.7) is the exponential term which rapidly decays to zero outside the circle $\alpha^2 + \beta^2 = k_1^2$ (also see Figure 14). This fact enables one to replace γ_2 , defined in (4.4b), by the first term of its Taylor's series expansion $\bar{\gamma}_2$, that is,

$$\gamma_2 = k_1 \sqrt{\kappa} \left[1 - \frac{\alpha^2 + \beta^2}{2k_1^2 \kappa} - \frac{1}{8} \left(\frac{\alpha^2 + \beta^2}{k_1^2 \kappa} \right)^2 - \dots \right] \quad ; \quad \alpha^2 + \beta^2 \leq k_1^2 |\kappa| \quad (4.8a)$$

$$\bar{\gamma}_2 = k_1 \sqrt{\kappa} \quad (4.8b)$$

For most practical values of κ , i.e., $|\kappa| \geq 10$, the approximation in (4.8b) is excellent inside the circle $\alpha^2 + \beta^2 \leq k_1^2$. Fortunately, as demonstrated in Figure 14, the decaying exponential in (4.7) can easily overcome the errors introduced by (4.8b) in the region $\alpha^2 + \beta^2 \geq k_1^2$ thereby making this a valid approximation throughout the $\alpha\beta$ -plane for a wide range of κ and z_2 parameters, namely,

$$\tilde{\Pi}_0 \approx f(\gamma_1, \bar{\gamma}_2) \exp(-jk_1 z_2) \quad (4.9)$$

where the bar on top represents the approximate quantities. It should be noted that Kuo and Mei [12, Eq. 8] have recently verified and used the aforementioned approximation for simplifying the space domain expressions,

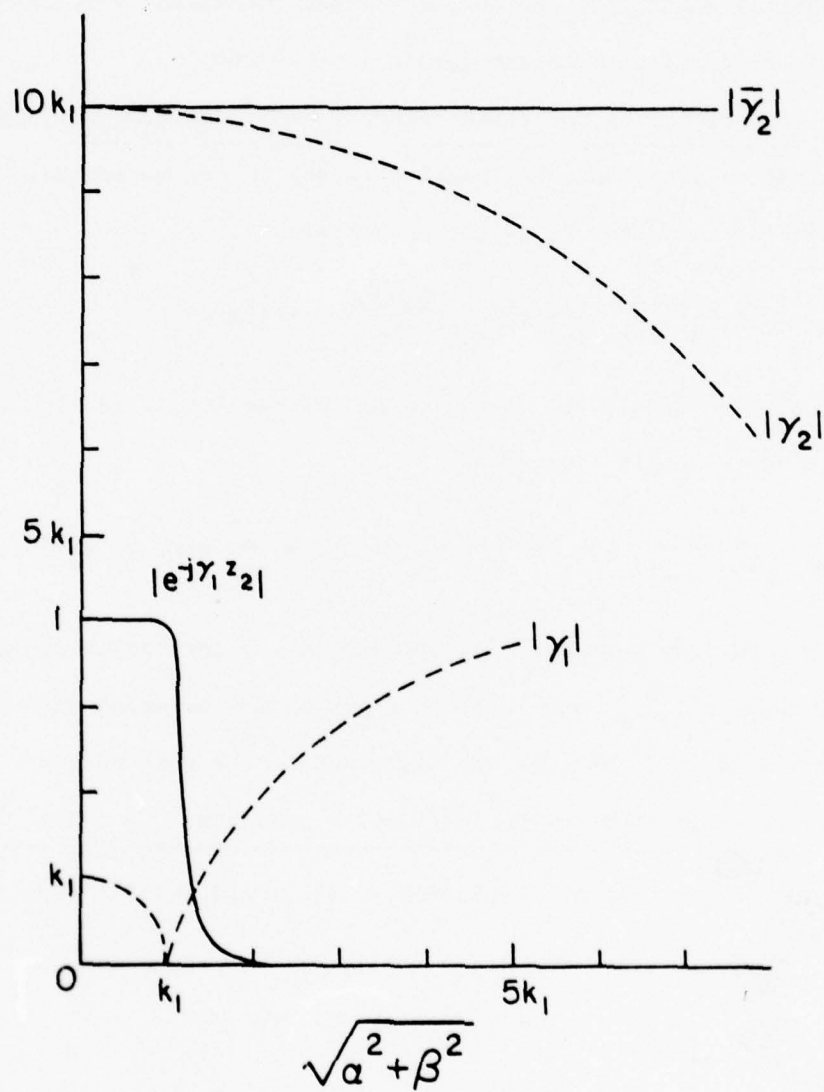


Figure 14. The plot of the functions $|\bar{\gamma}_2|$, $|\gamma_2|$, $|\gamma_1|$, and $|e^{-j\gamma_1 z_2}|$ versus $\sqrt{a^2 + \beta^2}$. Note that for $|\kappa|$ large enough, the following approximation holds: $\gamma_2 e^{-j\gamma_1 z_2} \approx \bar{\gamma}_2 e^{-j\gamma_1 z_2}$.

while Chang et al. [6] have employed an equivalent approximation for representing the vector potential in the vertical current element problem in terms of an incomplete Hankel function. The approximate transform domain vector potential quantities can now be treated individually in order to obtain their respective transform domain expressions.

4.2.1 Approximation for the horizontal component ($\tilde{\Pi}_{h1x}$). Applying the approximation introduced in Equation (4.8b) to the horizontal correction vector potential component in (4.1), one obtains

$$\tilde{\Pi}_{h1x} = I_{h0} \frac{1}{jk_1^2(1 - \kappa)} (\gamma_1 - k_1\sqrt{\kappa}) \exp(-j\gamma_1 z_2) \quad , \quad (4.10)$$

and direct application of the (4.6) identities results in the following closed-form space domain expression:

$$\tilde{\Pi}_{h1x} = \frac{-2I_{h0}}{k_1^2(1 - \kappa)} \left[jk_1\sqrt{\kappa} \frac{\partial}{\partial z} g(x, y, z_2) + \frac{\partial^2}{\partial z^2} g(x, y, z_2) \right] \quad . \quad (4.11)$$

Note that the various partials of g are explicitly derived in Appendix III. The useful range of $\tilde{\Pi}_{h1x}$ and the comparison of its accuracy and efficiency with other available techniques are discussed in the next section.

4.2.2 Approximation for the vertical components ($\tilde{\Pi}_{h1z}$ and $\tilde{\Pi}_{v1z}$).

Again the variable γ_2 can be eliminated in (4.2) and (4.3) by applying the approximation in (4.8b), that is,

$$\tilde{\Pi}_{h1z} = I_{h0} \frac{j\alpha}{k_1^2\kappa} \left[1 - j(\kappa + 1) \frac{c}{j(\gamma_1 + c)} \right] \exp(-j\gamma_1 z_2) \quad (4.12)$$

$$\tilde{\Pi}_{v1z} = I_{v0} \frac{1}{j(\gamma_1 + c)} \exp(-j\gamma_1 z_2) \quad , \quad (4.13)$$

where the constant c is defined as

$$c = k_1/\sqrt{\kappa} \quad . \quad (4.14)$$

Application of the identities in Equation (4.6) results in the following space domain forms:

$$0\bar{\Pi}_{hlz} = \frac{I_{h0}}{k_1^2} \left[-2 \frac{\partial^2}{\partial x \partial z} g(x, y, z_2) - jc(\kappa + 1) \frac{\partial}{\partial x} S(x, y, z_2) \right] \quad (4.15)$$

$$0\bar{\Pi}_{vlz} = I_{v0} S(x, y, z_2) \quad (4.16)$$

Unfortunately, the function S , whose transform domain expression is defined as

$$\tilde{S}(\alpha, \beta, z_2) = 0\bar{\Pi}_{vlz}/I_{v0} = \frac{1}{j(\gamma_1 + c)} \exp(-j\gamma_1 z_2) \quad (4.17)$$

cannot be expressed in a simple closed form. However, by using the (4.6) identities, it can be shown that S satisfies the following first-order inhomogeneous linear differential equation:

$$\frac{\partial}{\partial z} [S(\rho, z_2) \exp(-jcz_2)] = 2 \exp(-jcz_2) \frac{\partial}{\partial z} g(\rho, z_2) \quad (4.18)$$

where, for convenience, the functional dependences of S and g have been reduced from (x, y, z_2) to (ρ, z_2) . By integrating (4.18) with respect to z_2 , and after some algebraic manipulations, the following expression is derived for S :

$$S(\rho, z_2) = [S(\rho, z_2') - 2g(\rho, z_2')] \exp[jc(z_2 - z_2')] + 2g(\rho, z_2) + 2jc \exp(jcz_2) \int_{z_2'}^{z_2} g(\rho, z) \exp(-jcz) dz \quad (4.19)$$

The secondary height z_2' is an arbitrary starting point needed for obtaining a unique solution to the (4.18) differential equation. Since no simple starting point can be found at which $S(\rho, z_2')$ is known, it is assumed that z_2' is large enough so that the RCM approximation in Equation (3.27) is

applicable (see Figure 15), that is,

$$S(\rho, z'_2) = \frac{2\kappa \cos \theta'_2}{\kappa \cos \theta'_2 + \sqrt{\kappa - \sin^2 \theta'_2}} g(\rho, z'_2) \quad (4.20)$$

where angle θ'_2 is defined in Figure 15. A similar procedure can be used for computing $\frac{\partial}{\partial x} S$, required for $\bar{\Pi}_{hlz}$ in Equation (4.15), by simply differentiating (4.19) with respect to x :

$$\begin{aligned} \frac{\partial}{\partial x} S(\rho, z_2) = & \left[\frac{\partial}{\partial x} S(\rho, z'_2) - 2 \frac{\partial}{\partial x} g(\rho, z'_2) \right] \exp [jc(z_2 - z'_2)] + 2 \frac{\partial}{\partial x} g(\rho, z_2) \\ & + 2jc \exp (jcz_2) \int_{z'_2}^{z_2} \frac{\partial}{\partial x} g(\rho, z) \exp (-jcz) dz, \end{aligned} \quad (4.21)$$

and by using the asymptotic expression in (3.29), the following initial value at point z'_2 is obtained:

$$\begin{aligned} \frac{\partial}{\partial x} S(\rho, z'_2) = & \frac{2j}{c(\kappa + 1)} \left[\frac{\partial^2}{\partial x \partial z} g(\rho, z'_2) + k_1^2 \kappa \cos \phi_2 \sin \theta'_2 \cos \theta'_2 \right. \\ & \left. \cdot \frac{\cos \theta'_2 - \sqrt{\kappa - \sin^2 \theta'_2}}{\kappa \cos \theta'_2 + \sqrt{\kappa - \sin^2 \theta'_2}} g(\rho, z'_2) \right]. \end{aligned} \quad (4.22)$$

In summary, the procedure for computing the vertical vector potential components at the point (ρ, z_2) , as demonstrated in Figure 15, is to start from a higher observation point (ρ, z'_2) , at which the RCM expressions are valid, and simply use Equation (4.19) or (4.21) to integrate down to (ρ, z_2) . This procedure effectively computes a correction term for the RCM approximations of the vertical vector potential components in the region where RCM alone breaks down. The useful range and the accuracy of this procedure are discussed in the following section.

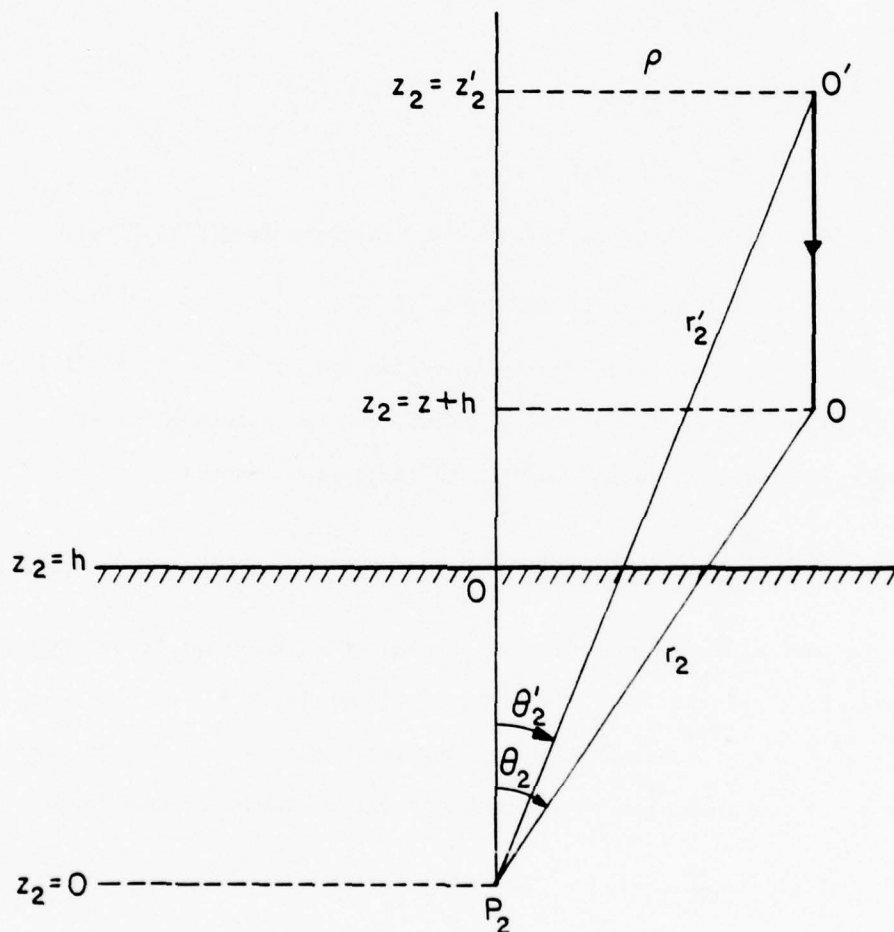


Figure 15. The geometry for computing the vertical vector potential components. r_2' is chosen to be large enough so that the RCM expressions are valid at O' . Therefore, the vector potential values along the interval $O'O$ are obtained by using the initial value at O' and integrating down along the z -axis.

4.3 Error Estimation for the Approximate Expressions

In the previous section, only the approximation in (4.8) was made to derive the horizontal vector potential component in Equation (4.11). The approximated term T in this vector potential component, for convenience, can be expressed in the following forms:

$$\tilde{T} = \gamma_2 \exp(-j\gamma_1 z_2) \quad (4.23a)$$

$$\tilde{\tilde{T}} = \bar{\gamma}_2 \exp(-j\gamma_1 z_2) \quad (4.23b)$$

$$\bar{T} = -2k_1 \sqrt{\kappa} \frac{\partial}{\partial z} g(\rho, z_2) \quad (4.23c)$$

The error introduced by (4.23b) in the transform domain is simply

$$\Delta \tilde{T} = \tilde{T} - \tilde{\tilde{T}} = (\gamma_2 - \bar{\gamma}_2) \exp(-j\gamma_1 z_2) \quad (4.24)$$

and again referring to the Taylor's series expansion for γ_2 in (4.8a), which is convergent for $\alpha^2 + \beta^2 < k_1^2 |\kappa|$, one can assume $\Delta \tilde{T}$ to be proportional to the second term of the expansion, namely,

$$\Delta \tilde{T} \approx \frac{-(\alpha^2 + \beta^2)}{2k_1 \sqrt{\kappa}} \exp(-j\gamma_1 z_2) \quad (4.25)$$

At the same time, a constraint is needed for z_2 in order to enforce the exponential term in (4.25) to decay sufficiently, i.e., to 1%, when $\alpha^2 + \beta^2 = k_1^2 |\kappa|$. This important condition, which allows one to use (4.25) for error analysis, can be expressed in the following simple form:

$$k_1 z_2 > \frac{5}{\sqrt{|\kappa|} - 1} \quad (4.26)$$

By substituting $\alpha^2 + \beta^2$ from (4.4a) into Equation (4.25), one can obtain ΔT directly via the (4.6) identities, namely,

$$\Delta T = \frac{k_1}{\sqrt{\kappa}} \frac{\partial}{\partial z} g(\rho, z_2) + \frac{1}{k_1 \sqrt{\kappa}} \frac{\partial^3}{\partial z^3} g(\rho, z_2) \quad (4.27)$$

The space domain expressions in (4.23c) and (4.27) can be used to define R_T , the relative error in the magnitude of \bar{T} , as

$$R_T = \left| \frac{\Delta T}{\bar{T}} \right| \leq \frac{1}{2|\kappa|} + \frac{1}{2k_1^2|\kappa|} \left| \frac{\frac{\partial^3}{\partial z^3} g(\rho, z_2)}{\frac{\partial}{\partial z} g(\rho, z_2)} \right| ; \quad (4.28)$$

and by using the explicit expansions provided in Appendix III, the above relation can be further reduced to

$$R_T < \frac{1}{2|\kappa|} + \frac{1}{2|\kappa|} \left\{ |5 \cos^2 \theta_2 - 3| \left[\frac{1}{(k_1 r_2)^2} \left(2 + \frac{1}{\sqrt{1 + (k_1 r_2)^2}} \right) + \frac{1}{k_1 r_2} \right] + \cos^2 \theta_2 \right\} . \quad (4.29)$$

Equation (4.29) is a useful upper bound for the error existing in the computed magnitude of $\bar{\Pi}_{hlx}$. The first term in the aforementioned equation sets a limit on the minimum required $|\kappa|$ value; for example, the error condition $R_T < 10\%$ requires $|\kappa| > 10$, while $R_T < 5\%$ will require $|\kappa| > 20$. Figures 16 and 17 demonstrate the valid regions in the (r_2, θ_2) plane as a function of $|\kappa|$ for two useful error conditions, namely, $R_T = 5\%$ and 10% , respectively. Table 4.1 is included to numerically verify the conditions (4.26) and (4.29) by comparing the approximate horizontal vector potential values in (4.11) with the corresponding exact SDP integration results of Chapter 3. As clearly demonstrated in this table, the actual computed errors in the magnitude are consistently below the R_T values obtained from Equation (4.29), for the various (κ, r_2, θ_2) combinations tested.

Unfortunately, because of the complicated nature of the approximate transform expressions (4.12) - (4.13), one cannot derive a simple error condition for the vertical potential components, as was derived for the horizontal case (Equation (4.29)). However, because of the similar nature

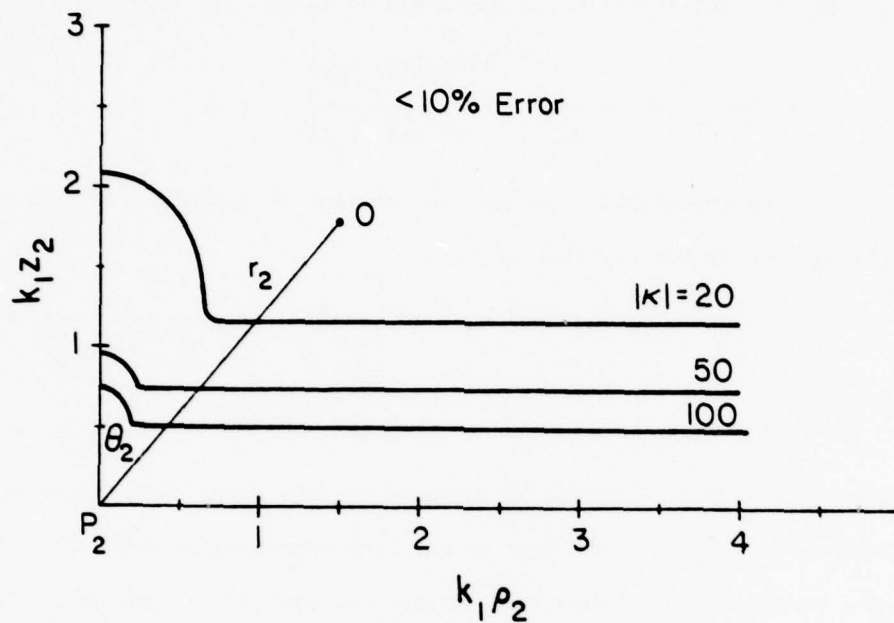


Figure 16. Examples of $|\kappa|$ contours for which $< 10\%$ error is ensured for observation points on or above it.

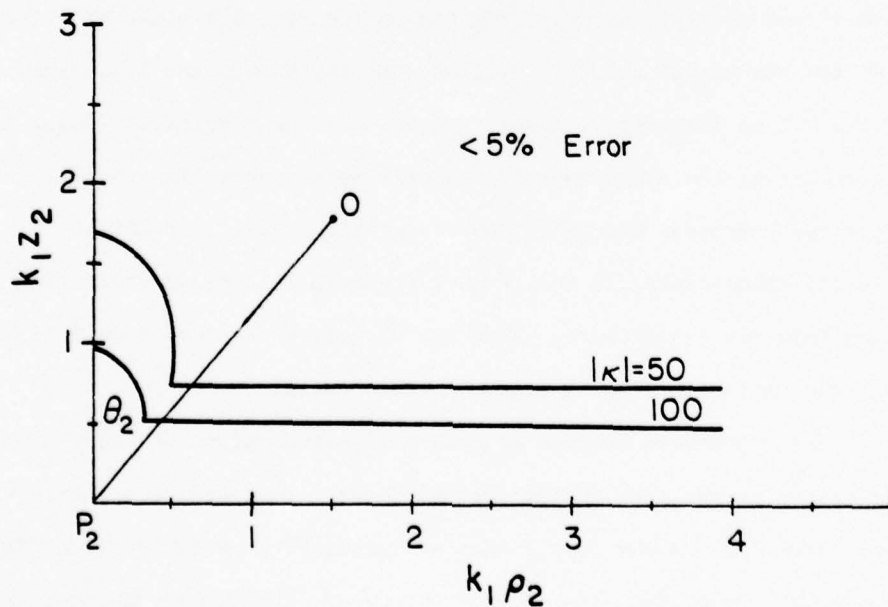


Figure 17. Examples of $|\kappa|$ contours for which $< 5\%$ error is ensured for observation points on or above it.

of the approximation for all three of the correction vector potential components, one can assume that conditions (4.26) and (4.29) also apply to the $\bar{\Pi}_{0hlz}$ and $\bar{\Pi}_{0vlz}$ expressions shown in Equations (4.15) - (4.16). This assumption is verified by the results in Tables 4.2 and 4.3 in which the vertical-correction-vector potential components are computed by using $k_1 r_2' = 10$ as a starting point. Even though these RCM initial values, in general, contain about 5% error, the net errors in all of the cases are well below the predicted ones (Equation (4.29)). Table 4.4 is also included to demonstrate the stability of the final vertical vector potential value as a function of starting point z_2' . Note that when $k_1 r_2' > 10$, the error in the final results stays within acceptable levels of the exact values primarily because the RCM approximation used to compute the starting point becomes reasonably accurate in this region.

Studying the results of this section, one can conclude that the proposed approximate formulas, i.e., (4.11), (4.15) and (4.16), can be employed in most practical antenna problems as long as $|k| > 10$ and condition (4.26) is satisfied.

4.4 Advantages of the Approximate Expressions

The major advantage of the approximate technique introduced in the chapter is, of course, the computation of the vector potentials. For example, on the Cyber-175 computer, the computation time for computing the three vector potential expressions is ≈ 1 msec for the RCM technique, ≈ 5 msec for the approximate technique presented in this chapter, and finally ≈ 50 -100 msec for the SDP integration technique discussed in Chapter 3. Note that even though the approximate technique is slightly more time-consuming than the RCM method, it can be applied to a much

Table 4.1 Comparison of the RCM, exact, and the approximate evaluations of ϕ_{v12} . For this example, $f = 30$ MHz, $\theta_2 = 45^\circ$, $\phi_2 = 0^\circ$, and $I_{v0} = 1$.

$k_1 r_2$	$k_1 z_2$	Min. $k_1 z_2$ Eq. (4.26)	RCM value $\times 10^2$	Exact integration $\times 10^2$	Approximate technique $\times 10^2$	% error Eq. (4.29)	% error actual
$\epsilon = 5, \alpha = .01, \kappa = 11.7$							
.1	.07	2.49	62.1-j7.49	79.6-j11.2	96.3-j17.2	1551.	16.0
1.	.71	2.49	3.28-j5.33	3.22-j6.52	2.94-j7.13	33.3	8.7
2.	1.41	2.49	-1.36-j2.82	-1.84-j2.95	-1.96-j2.88	20.4	4.0
6.	4.24	2.49	1.01+j.272	1.06+j2.00	1.03-j2.09	16.0	2.9
10.	7.07	2.49	-.518+j.351	-.507+j.383	-.518+j.351	15.5	5.5
$\epsilon = 10, \alpha = .01, \kappa = 10$							
.1	.07	1.53	70.5-j12.4	90.8-j15.8	96.6-j15.7	67.0	5.9
1.	.71	1.53	3.41-j6.30	3.47-j7.76	3.42-j7.86	14.4	1.2
2.	1.41	1.53	-1.73-j3.14	-2.23-j3.34	-2.28-j3.30	8.8	1.7
6.	4.24	1.53	1.17+j.247	1.23+j.184	1.19+j.179	6.9	2.8
10.	7.07	1.53	-.570+j.433	-.562+j.465	-.570+j.433	6.7	4.6
$\epsilon = 601, \alpha = 1, \kappa = 601$							
.1	.07	.204	95.0-j13.2	99.5-j11.0	99.6-j11.0	13.0	.05
1.	.71	.204	4.87-j8.26	5.09-j8.52	5.09-j8.52	.28	.06
2.	1.41	.204	-2.16-j4.28	-2.22-j4.39	-2.22-j4.40	.17	.10
6.	4.24	.204	1.55+j.388	1.57+j.386	1.57+j.386	.13	.35
10.	7.07	.204	-.784+j.552	-.788+j.560	-.784+j.552	.13	.95

Table 4.2 Comparison of the RCM, exact, and the approximate evaluations of Π_{hlx} . For this example, $f = 30$ MHz, $\theta_2 = 45^\circ$, $\phi_2 = 0^\circ$, and $I_{h0} = 0$.

k_1, r_2	k_1, z_2	Min. k_1, z_2 Eq. (4.26)	RCM value $\times 10^2$	Exact integration $\times 10^2$	Approximate technique $\times 10^2$	% error Eq. (4.29)	% error actual
.1	.07	2.49	24.9-j1.25	48.4-j7.82	1251.-j209.	1551.	96.1
1.	.71	2.49	1.45-j2.03	1.11-j3.62	-.892+j1.51	33.3	29.4
2.	1.41	2.49	-.462-j1.16	-.996-j1.18	-1.15-j1.16	20.4	9.7
6.	4.24	2.49	.393+j1.36	.420+j.078	.452+j.084	16.0	7.1
10.	7.07	2.49	-.216+j.125	-.205+j.146	-.222+j.156	15.5	7.3
<hr/>							
.1	.07	1.53	17.4+j.023	45.6-j9.38	452.6+j42.2	670.	90.3
1.	.71	1.53	1.26-j1.22	.712-j2.55	.510-j2.72	14.4	9.4
2.	1.41	1.53	-.171-j.859	-.576-j.847	-.592-j.858	8.8	1.9
6.	4.24	1.53	.254+j.144	.278+j.105	.284+j.110	6.9	2.6
10.	7.07	1.53	-.165+j.059	-.159+j.075	-.164+j.074	6.7	2.7
<hr/>							
.1	.07	.204	-2.24-j1.52	-14.5+j5.41	-21.5-j3.74	13.0	9.8
1.	.71	.204	.272-j.074	.206-j.342	.205-j.343	.28	.09
2.	1.41	.204	.042-j1.35	-.024-j1.56	-.024-j1.56	.17	.03
6.	4.24	.204	.025+j.040	.031+j.036	.031+j.036	.13	.03
10.	7.07	.204	-.028-j.004	-.028-j.002	-.028-j.002	.13	.04

Table 4.3 Comparison of the RCM, exact, and the approximate evaluations of θ_{hlz} . For this example, $f = 30$ MHz, $\theta_2 = 45^\circ$, $\phi_2 = 0^\circ$, and $I_{v0} = 0$.

$k_1 r_2$	$k_1 z_2$	Min. $k_1 z_2$ Eq. (4.26)	RCM value $\times 10^2$	Exact integration $\times 10^2$	Approximate technique $\times 10^2$	% error Eq. (4.29)	% error actual
.1	.07	2.49	-12.5+j1.25	-14.3+j1.19	-2984.+j23.0	1551.	99.5
1.	.71	2.49	-.677+j1.05	-.892+j1.51	-1.94+j4.14	33.3	61.9
2.	1.41	2.49	.260+j.570	.463+j.699	.683+j.915	20.4	27.0
6.	4.24	2.49	-2.01-j.058	-.225-j.027	-.238-j.032	16.0	5.6
10.	7.07	2.49	.105-j.068	.100-j.082	.105-j.068	15.5	12.7
.1	.07	1.53	-11.6-j.126	-18.2+j2.63	-1170.-j448.5	670.	98.7
1.	.71	1.53	-.753+j.903	-.676+j1.65	-1.16+j2.24	14.4	30.3
2.	1.41	1.53	.182+j.552	.445+j.611	.456+j.680	8.8	8.6
6.	4.24	1.53	-.179-j.074	-.202-j.043	-.199-j.049	8.8	3.2
10.	7.07	1.53	.104-j.052	.997-j.066	.104-j.052	6.7	12.8
.1	.07	.204	-2.24-j1.52	-14.5+j5.41	-21.5-j3.74	13.0	9.8
1.	.71	.204	-.258+j.082	-.204+j.328	-.205+j.328	.28	.33
2.	1.41	.204	-.035+j1.31	.026+j1.53	.026+j1.53	.17	.42
6.	4.24	.204	-.025-j.037	.032-j.034	-.032-j.032	.13	4.1
10.	7.07	.204	.027+j.003	.027+j.0005	.027+j.003	.13	10.4

Table 4.4 Demonstration of the stability of the approximate technique as a function of secondary height z'_2 . In this example, $f = 18$ MHz, $r_2/\lambda = 0.6$, and $\theta_2 = 45^\circ$.

z_2/λ	z'_2/λ	Approximate $0^{\text{II}}_{\text{h1z}} \times 10^3$
.42	.50	1.82-j.953
.42	.75	1.76-j1.39
.42	1.00	1.57-j1.31
.42	1.25	1.64-j1.22
.42	1.50	1.68-j1.29
.42	1.75	1.63-j1.30
.42	2.00	1.62-j1.25

At $z_2/\lambda = .42$ RCM $0^{\text{II}}_{\text{h1z}} \times 10^3 = 1.64-j.773$

At $z_2/\lambda = .42$ Exact $0^{\text{II}}_{\text{h1z}} \times 10^3 = 1.58-j1.29$

wider range of parameters making it suitable for most practical antenna problems.

Another advantage of this technique is that the horizontal vector potential component $\bar{\Pi}_{hlx}$ is expressed in a simple closed form (Equation 4.11), while Kuo and Mei [12] have obtained yet another infinite integral form by applying virtually the same approximation as the one in Equation (4.8b) to the Sommerfeld integrals. Also the finite integrations needed for the vertical vector potential approximations, viz., Equations (4.19) and (4.21), will never have a singularity in their integration interval thus making the integrands well-behaved. In addition, these integrands are independent of z'_2 , thereby making it possible to compute the vertical vector potential values along the vertical interval $z'_2 z_2$ by a single integration, i.e., by using the newly computed value of the vector potential as an initial value for computing the next point on the interval. This procedure, when needed, can appreciably improve the overall efficiency of the technique.

Finally, since the evaluation of the E- and the H-fields is of major importance, the approximate technique presented in this chapter also provides a computationally efficient formulation for computing the various field components. This can be simply demonstrated by observing the matrix Equations (2.9) and (2.10). The various mixed partial derivatives of Π_x and Π_z , needed for the field computations, can be easily expressed in terms of the various mixed partial derivatives of $g(\rho, z_2)$, which in turn are expressed in a computationally efficient closed form in Appendix III.

The following chapter demonstrates the ability of these approximate expressions to efficiently analyze several antenna structures radiating over a lossy half-space.

5. WIRE ANTENNAS RADIATING OVER A LOSSY HALF-SPACE

The approximate field solution to the current element problem radiating over a lossy half-space, developed in the previous chapter, in conjunction with the method of moments [17] can be employed to analyze a wide variety of thin-wire antenna problems radiating over a lossy half-space. Initially, in this chapter, a general integral equation is derived containing the unknown antenna current. The method of moments is then used to reduce the integral equation into a numerically manageable matrix form, and finally, a digital computer program is developed for computing the antenna currents, impedance, and far-field patterns, given a specified antenna geometry. A number of simple antenna structures are considered and their behaviours are numerically predicted and presented in the final sections.

5.1 Antenna Integral Equation

Figure 18 depicts the geometry of an arbitrary wire antenna over a lossy half-space, with (r_a, θ_a, ϕ_a) defining a point on the antenna-axis. For simplicity, it is assumed that the antenna is entirely in the xz-plane ($\phi_a = 0$), since the field computation due to currents in the y-direction can easily be handled by a digital computer program by simply rotating the xy-plane by 90° about the z-axis. Assuming that the antenna is excited by the field $\vec{E}^{\text{exc}}(\vec{r}_a)$, and having a loading function $\Lambda(\vec{r}_a)$ ohms/meter, one can write a general integral equation enforcing the total E-field along the antenna equal to that induced by the possible loading function, that is,

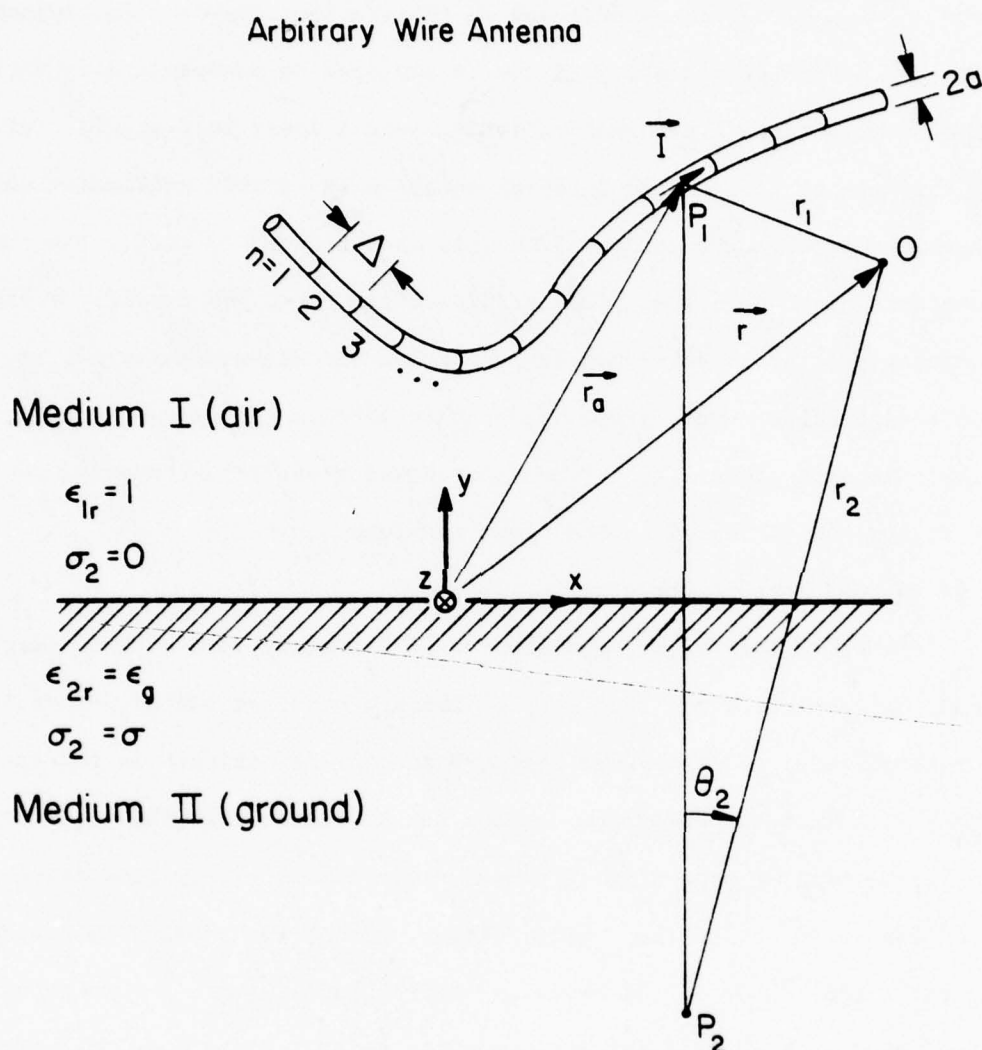


Figure 18. The geometry of an arbitrarily shaped wire antenna located over a lossy half-space.

$$\begin{aligned} \hat{\mathbf{I}}(\vec{r}_a) \cdot \vec{\mathbf{E}}^{\text{exc}}(\vec{r}_a) + \hat{\mathbf{I}}(\vec{r}_a) \cdot \int_{\text{antenna}} \left[\vec{\mathbf{G}}_v(\vec{r}_a, \vec{r}'_a) \hat{\mathbf{z}} \cdot \vec{\mathbf{I}}(\vec{r}'_a) + \vec{\mathbf{G}}_h(\vec{r}_a, \vec{r}'_a) \hat{\mathbf{x}} \cdot \vec{\mathbf{I}}(\vec{r}'_a) \right] d\vec{r}'_a \\ = \Lambda(\vec{r}_a) \vec{\mathbf{I}}(\vec{r}_a) \quad , \end{aligned} \quad (5.1)$$

where $\hat{\mathbf{I}}(\vec{r}_a)$ is a unit vector along the antenna direction and the kernels $\vec{\mathbf{G}}_v(\vec{r}_a, \vec{r}'_a)$ and $\vec{\mathbf{G}}_h(\vec{r}_a, \vec{r}'_a)$ are the E-fields induced at point \vec{r}_a due to a one-ampere electric current element located at \vec{r}'_a and oriented in the z- and the x-directions, respectively. The matrix Equation (2.9), which formulates the electric-field components in terms of the vector potentials, is used to write:

$$\vec{\mathbf{E}} = \vec{\mathbf{D}}_x \Pi_x + \vec{\mathbf{D}}_z \Pi_z \quad , \quad (5.2)$$

where the vector operators $\vec{\mathbf{D}}_x$ and $\vec{\mathbf{D}}_z$ are defined as:

$$\vec{\mathbf{D}}_x = \hat{\mathbf{x}} \left(k_1^2 + \frac{\partial^2}{\partial x^2} \right) + \hat{\mathbf{y}} \left(\frac{\partial^2}{\partial x \partial y} \right) + \hat{\mathbf{z}} \left(\frac{\partial^2}{\partial x \partial z} \right) \quad (5.3a)$$

$$\vec{\mathbf{D}}_z = \hat{\mathbf{x}} \left(\frac{\partial^2}{\partial x \partial z} \right) + \hat{\mathbf{y}} \left(\frac{\partial^2}{\partial y \partial z} \right) + \hat{\mathbf{z}} \left(k_1^2 + \frac{\partial^2}{\partial z^2} \right) \quad . \quad (5.3b)$$

By using the general formula in (5.2), one can directly write the two kernels, $\vec{\mathbf{G}}_v$ and $\vec{\mathbf{G}}_h$, in terms of the incident, perfect reflection, and the correction vector potential components (see Equations (2.20), (2.30), and (2.32)), namely,

$$\vec{\mathbf{G}}_v(\vec{r}_a, \vec{r}'_a) = \vec{\mathbf{G}}_v^i(\vec{r}_a, \vec{r}'_a) + \vec{\mathbf{G}}_v^r(\vec{r}_a, \vec{r}'_a) + {}_0\vec{\mathbf{G}}_v(\vec{r}_a, \vec{r}'_a), \quad (5.4)$$

where

$$\vec{\mathbf{G}}_v^i(\vec{r}_a, \vec{r}'_a) = (j\omega\epsilon_0)^{-1} \vec{\mathbf{D}}_z g(\vec{r}_a - \vec{r}'_a) \quad (5.5a)$$

$$\vec{\mathbf{G}}_v^r(\vec{r}_a, \vec{r}'_a) = (j\omega\epsilon_0)^{-1} \vec{\mathbf{D}}_z g(\vec{r}_a - \vec{r}'_a - 2h\hat{\mathbf{z}}) \quad (5.5b)$$

$${}_0\vec{\mathbf{G}}_v(\vec{r}_a, \vec{r}'_a) = \vec{\mathbf{D}}_z {}_0\Pi_{v1z}(\vec{r}_a - \vec{r}'_a - 2h\hat{\mathbf{z}}) \Big|_{I_v = 1} \quad , \quad (5.5c)$$

and

$$\vec{G}_h(\vec{r}_a, \vec{r}_a') = \vec{G}_h^i(\vec{r}_a, \vec{r}_a') + \vec{G}_h^r(\vec{r}_a, \vec{r}_a') + {}_0\vec{G}_h(\vec{r}_a, \vec{r}_a') \quad , \quad (5.6)$$

where

$$\vec{G}_h^i(\vec{r}_a, \vec{r}_a') = (j\omega\epsilon_0)^{-1} \vec{D}_x g(\vec{r}_a - \vec{r}_a') \quad (5.7a)$$

$$\vec{G}_h^r(\vec{r}_a, \vec{r}_a') = (j\omega\epsilon_0)^{-1} \vec{D}_x g(\vec{r}_a - \vec{r}_a' - 2h\hat{z}) \quad (5.7b)$$

$${}_0\vec{G}_h(\vec{r}_a, \vec{r}_a') = \left[\vec{D}_x {}_0\Pi_{h1x}(\vec{r}_a - \vec{r}_a' - 2h\hat{z}) + \vec{D}_z {}_0\Pi_{h1z}(\vec{r}_a - \vec{r}_a' - 2h\hat{z}) \right] \quad . \quad (5.7c)$$

$I_h = 1$

Note that in the above equations, $h = r_a' \cos(\theta_a')$ is the height of the current source above the half-plane interface and g is the free-space Green's function defined in (4.5b).

The correction vector potential formulation of Chapter 4, namely, Equations (4.11), (4.15), and (4.16), along with the expansions presented in Appendix III, enable one to compute the scattered components of the two kernels, viz., \vec{G}_v^r , \vec{G}_v^i , \vec{G}_h^r , and ${}_0\vec{G}_h$, without difficulty. However, because of the singular nature of g , the free-space solution of the kernels, \vec{G}_v^i and \vec{G}_h^i , should not be computed directly. Instead, as has been successfully reported [11], [14], [18], and [19], the thin-wire approximation is used to shift the observation point \vec{r}_a to the antenna surface, and in order to further smooth out the singularities, the finite difference scheme is employed to perform the \vec{D}_x and \vec{D}_z operations defined in (5.3).

5.2 Method of Moments

As developed by Harrington [17], the method of moments is a convenient approximation for transforming the antenna integral equation into a numerically manageable matrix form. In this work, pulse-basis and delta-matching functions

are chosen since they eliminate the need for integrating the kernels \vec{G}_v and \vec{G}_h . The number of unknown patches on the antenna N should be large enough so that the patch length Δ is at most $1/6$ of the wavelength. The approximated current along the antenna is therefore represented as:

$$\vec{I} = \sum_{n=1}^N I_n \hat{I}_n, \quad (5.8)$$

for which I_n is an unknown constant value over the n^{th} patch and zero outside of it, also \hat{I}_n is a known unit vector tangent to the antenna at the center of the n^{th} patch (see Figure 18). Substituting (5.8) into (5.1) and letting subscripts $n = 1, 2, 3, \dots$ denote "evaluation at the center of the n^{th} patch," and letting $[\vec{I}]$ and $[\vec{E}^{\text{exc}}]$ be column vectors containing the current and the tangential excitation field values at successive patches, one finally arrives at

$$[\vec{E}^{\text{exc}}] = -[Z^{\text{imp}}][\vec{I}] + [\Lambda][\vec{I}], \quad (5.9)$$

where $[\Lambda]$ is a diagonal matrix with elements $\Lambda_1, \Lambda_2, \dots, \Lambda_n$; and $[Z^{\text{imp}}]$ is an $n \times n$ square matrix with its i^{th} row and j^{th} column element defined as:

$$Z_{ij}^{\text{imp}} = \Delta \hat{I}(\vec{r}_{ai}) \cdot [\hat{z} \cdot \hat{I}(\vec{r}_{aj}) \vec{G}_v(\vec{r}_{ai}, \vec{r}_{aj}) + \hat{x} \cdot \hat{I}(\vec{r}_{aj}) \vec{G}_h(\vec{r}_{ai}, \vec{r}_{aj})] \quad (5.10)$$

Note that i and j also refer to patch numbers, and Δ is the patch length.

The matrix Equation (5.9) can be solved for the unknown currents $[\vec{I}]$, and by replacing the excitation E-field in terms of the excitation voltage $[\vec{V}]$, one arrives at

$$[\vec{I}] = [Y^{\text{ant}}][\vec{V}] \quad (5.11a)$$

$$[Y^{\text{ant}}] = \Delta^{-1} \{-[Z^{\text{imp}}] + [\Lambda]\}^{-1} \quad (5.11b)$$

Once $[Y^{\text{ant}}]$ is constructed for a given structure, the antenna currents can

be directly computed for a given voltage source excitation. As was discussed in the previous section, the thin-wire approximation and the finite-difference scheme are employed for evaluating the free-space solution components of the kernels, viz., \vec{G}_v^i and \vec{G}_h^i . A power series, derived by Harrington [20], is used for the thin-wire approximation computations, and based on the conclusions made in [11], [14], [18], and [19], the finite difference parameter δ is chosen to be equal to $\Delta/2$, e.g.,

$$\left. \frac{\partial^2 f}{\partial x^2} \right|_{x_0} \approx \frac{f(x_0 + \delta) + f(x_0 - \delta) - 2f(x_0)}{\delta^2} ; \quad \delta = \Delta/2 . \quad (5.12)$$

The free-space solution obtained by using the above approach has been thoroughly tested and, as an example, the generated impedance curves shown in Figures 19 and 20 agree well with the ones reported by Jordan et al. [21].

5.3 Far-Field Radiation Pattern

The RCM expressions, shown in Equations (3.27) - (3.29), are the logical choices for representing the correction vector potential components in the far-field region ($k_1 r \gg 10$). Working in the spherical coordinate system (r, θ, ϕ) , and neglecting all terms containing r^{-2} , r^{-3} , ..., one can easily show

$$\nabla \nabla \cdot \vec{\Pi} \approx -k^2 \Pi_r \hat{r} , \quad (5.13)$$

where $\vec{\Pi}$ is the total vector potential. The total electric field \vec{E} , from Equation (2.8), can therefore be shown to have no \hat{r} -component, namely,

$$\vec{E}(r, \theta, \phi) = (k^2 + \nabla \nabla \cdot) \vec{\Pi} = k^2 [(\cos \theta \cos \phi \Pi_x - \sin \theta \Pi_z) \hat{\theta} - \sin \phi \Pi_x \hat{\phi}] . \quad (5.14)$$

As expected, the above far-field expression represents two plane waves

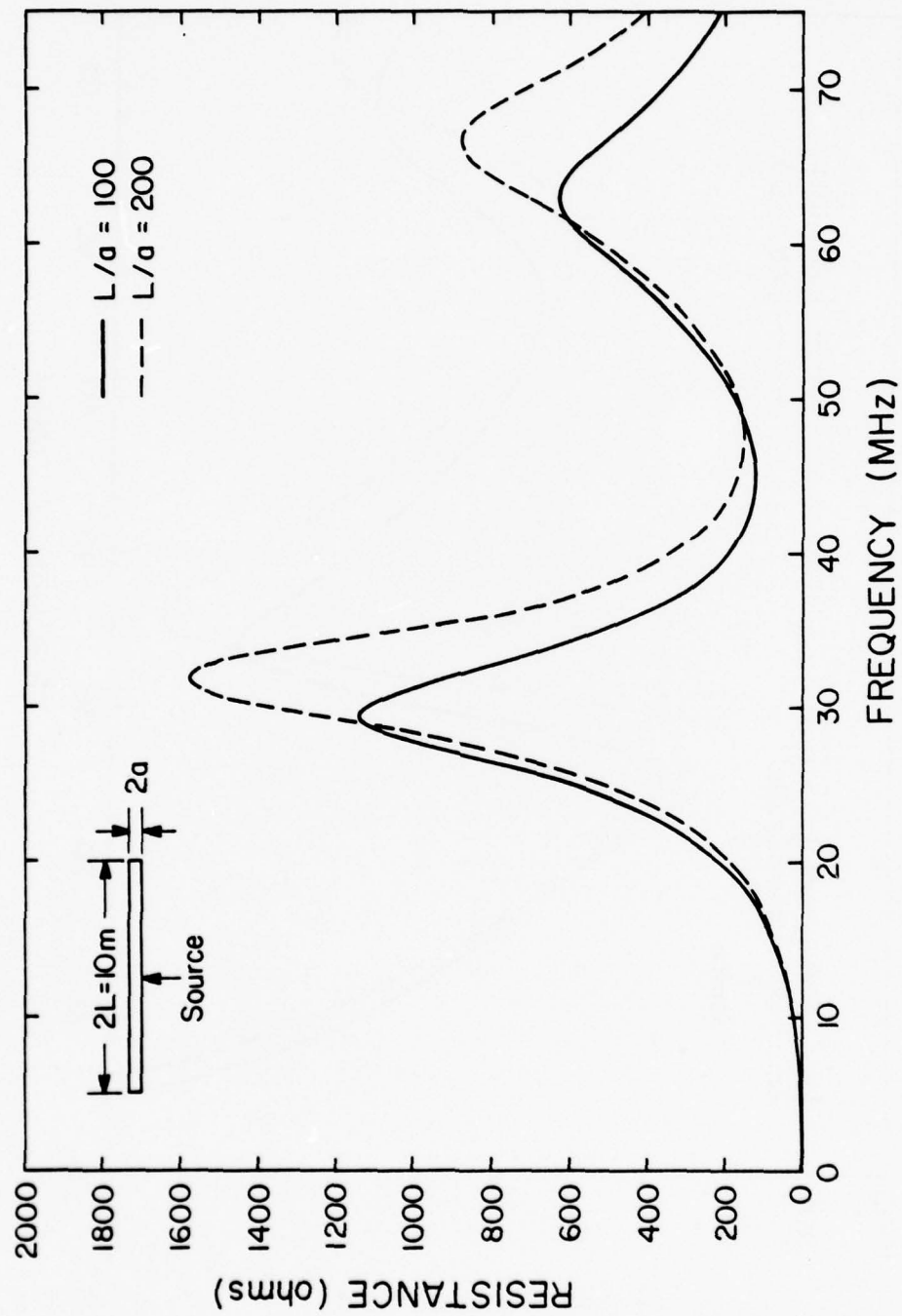


Figure 19. Input resistance of an unloaded dipole antenna ($2L = 10m$) radiating in free space.

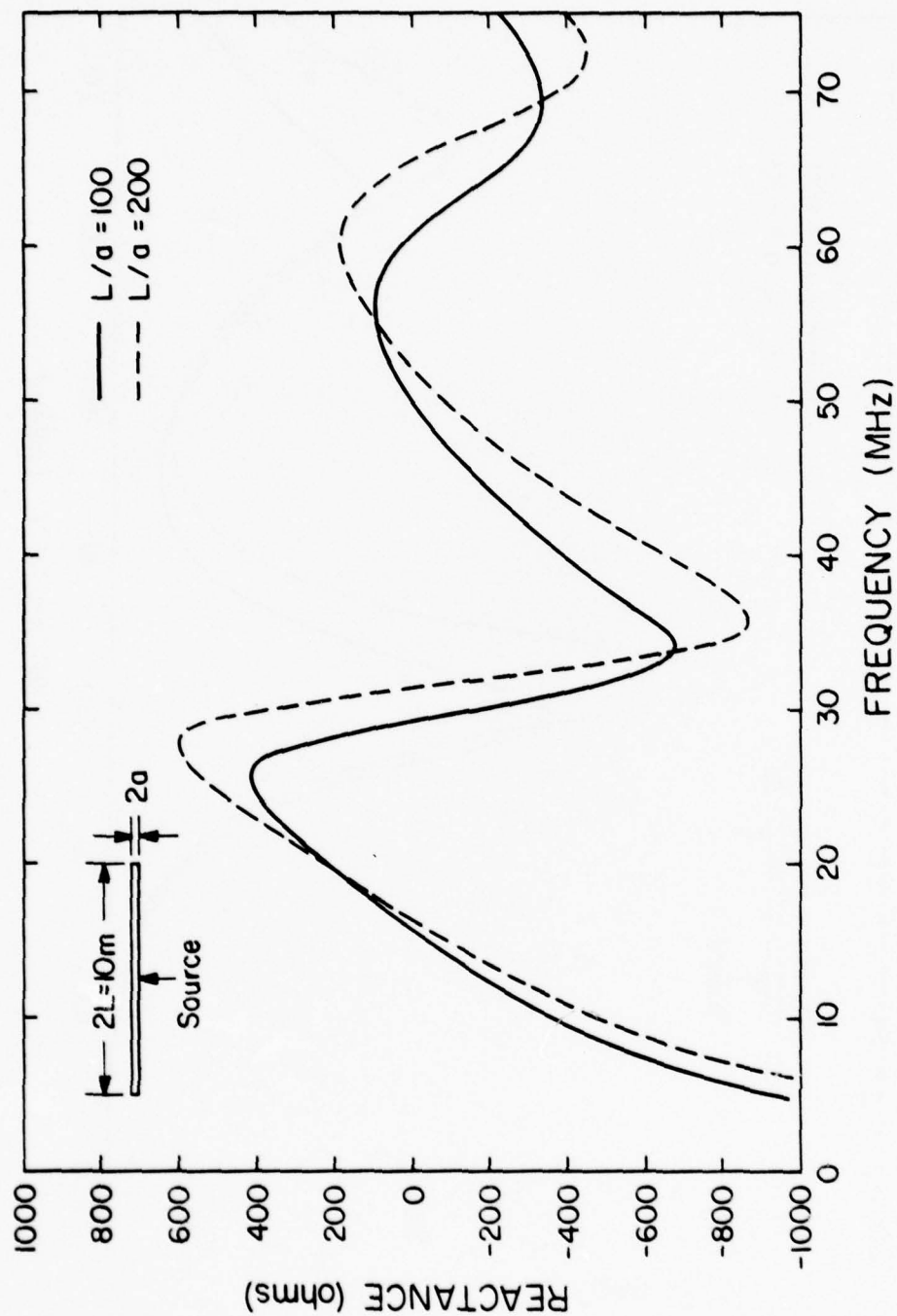


Figure 20. Input reactance of an unloaded dipole antenna ($2L = 10m$) radiating in free space.

(polarized in the $\hat{\theta}$ - and the $\hat{\phi}$ - directions) propagating away from the (x,y,z) origin defined in Figure 18. In summary, the far-electric-field radiation pattern due to a current element with no \hat{y} -component can be readily computed by initially evaluating the total vector potential components via the RCM approximation and then using Equation (5.14) to obtain the E-field values at the desired observation points.

The far-field pattern for a given antenna structure is simply obtained by applying the superposition theorem to the individual radiation patterns of the antenna current segments defined in the method of moments approximation. Radiation pattern examples are included for the various antenna structures analyzed in the following sections.

5.4 Horizontal Antenna over Lossy Half-Space

The general developments of Sections 5.1 - 5.3 are applied to the horizontal antenna shown in Figure 21. Fortunately, because of the symmetries present in this geometry, the $[Z^{imp}]$ matrix in Equation (5.9) takes the following form (Toeplitz matrix):

$$[Z^{imp}] = \begin{bmatrix} a_1 & a_2 & a_3 & \cdot & \cdot & \cdot & a_n \\ a_2 & a_1 & a_2 & \cdot & \cdot & \cdot & a_{n-1} \\ a_3 & a_2 & a_1 & \cdot & \cdot & \cdot & a_{n-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_n & a_{n-1} & a_{n-2} & \cdot & \cdot & \cdot & a_1 \end{bmatrix} \quad (5.15)$$

Therefore, one needs to compute only one row of this matrix and use the aforementioned symmetry to complete it. The main program, HORIZ (see Appendix IV),

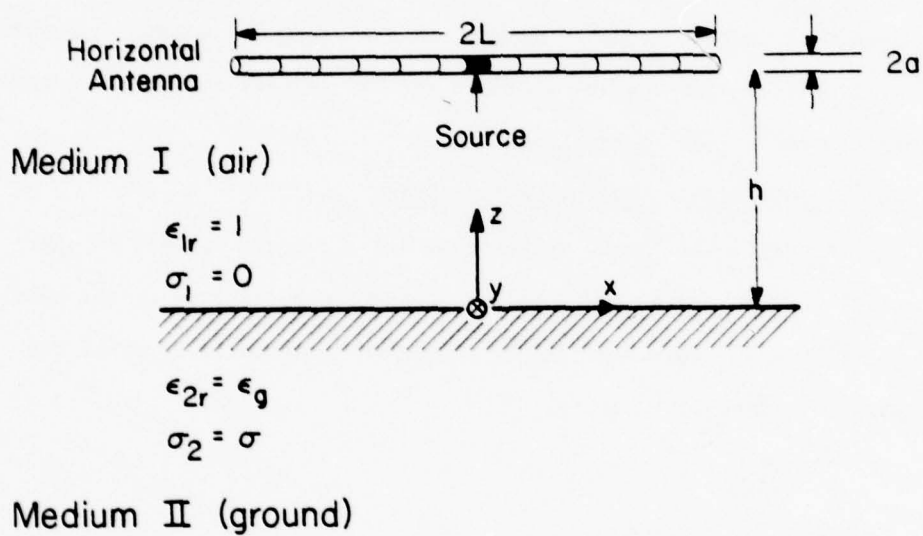


Figure 21. Center-fed horizontal dipole over a lossy half-space.

is developed to analyze the horizontal antenna of Figure 21. Using this program, Figures 22 and 23 are generated to show the impedance variations of a $2L = 10$ meters center-fed horizontal antenna located $h = 3$ meters above various lossy grounds. Radiation pattern of this antenna at 15 MHz is also shown in Figures 24 and 25.

5.5 Vertical Antenna over Lossy Half-Space

The vertical dipole shown in Figure 26 is the next geometry considered. Unfortunately, as was the case for the horizontal antenna, the total $[Z^{imp}]$ matrix is not in a Toeplitz form. However, main program VERT (in Appendix IV) is designed to take maximum advantage of the available symmetry. As an example, Figure 27 is included to show the radiation pattern of a $2L = 10$ meters, $h = 8$ meters, center-fed vertical dipole at resonance ($f = 15$ MHz) located over various lossy grounds.

5.6 Inverted Vee-Dipole

As a complicated example, the inverted Vee-dipole of Figure 28 is considered. Again, as in the two previous sections, symmetry is used in the main program VEEDIP (Appendix IV) in constructing the $[Z^{imp}]$ matrix. The program is tested for an inverted Vee-dipole structure having $L = 7.5$ meters, $h = 10$ meters, and $\psi = 90^\circ$; Figures 29 and 30 demonstrate the radiation pattern of this structure at 10 MHz and for various lossy grounds.

In all three of these examples, care has been taken not to violate the conditions $|\kappa| > 10$ and Equation (4.26) to ensure the accuracy of the results. Also, since the $[Z^{imp}]$ matrix for these examples turns out to be symmetric, a special inversion routine (XINVZ in Appendix IV) is employed to save an appreciable amount of computer time.

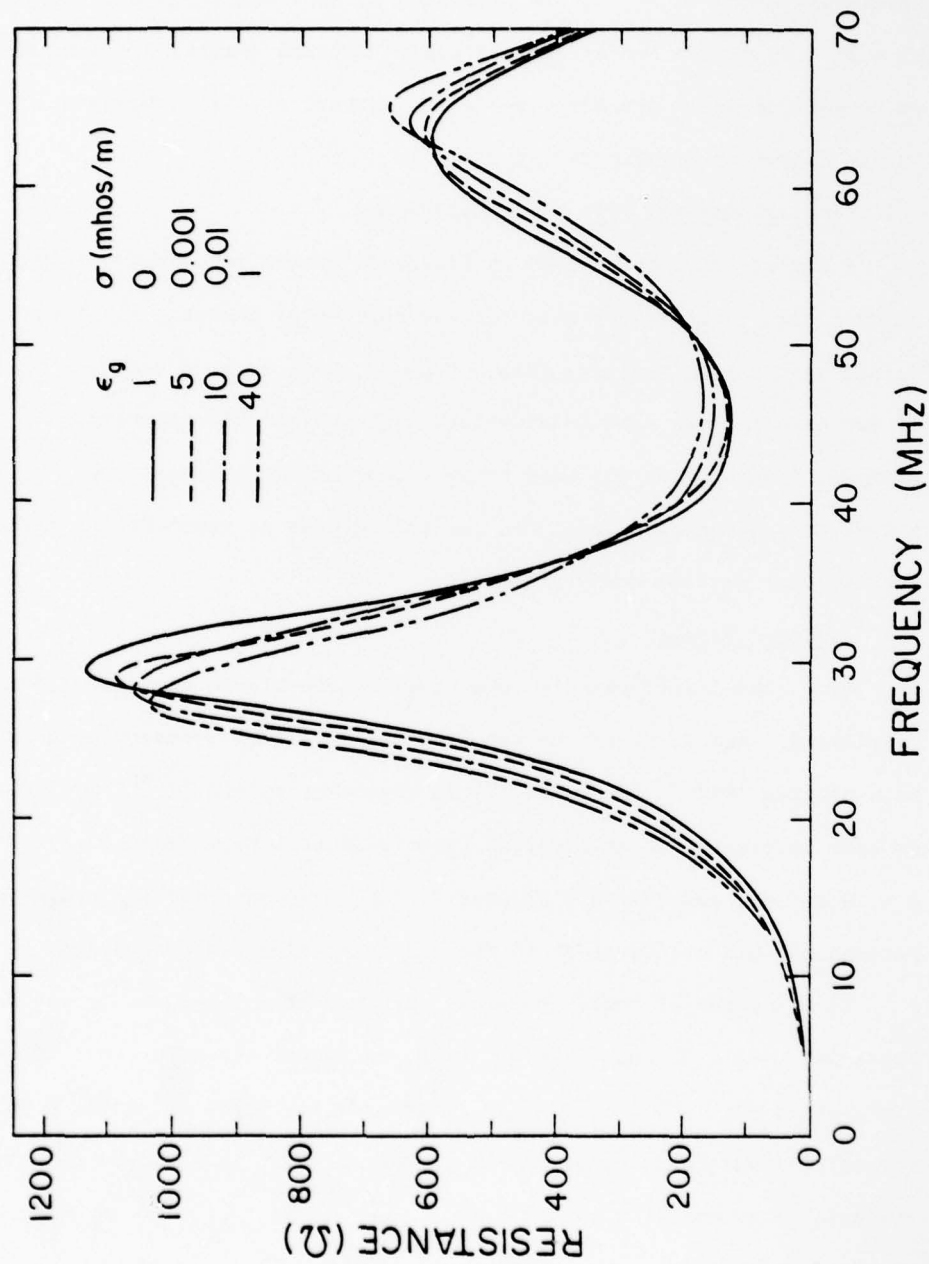


Figure 22. Input resistance of a center-fed horizontal dipole antenna as a function of frequency and the ground parameters. Note that $2L = 10\text{m}$, $2a = 0.1\text{m}$, and $h = 3\text{m}$.

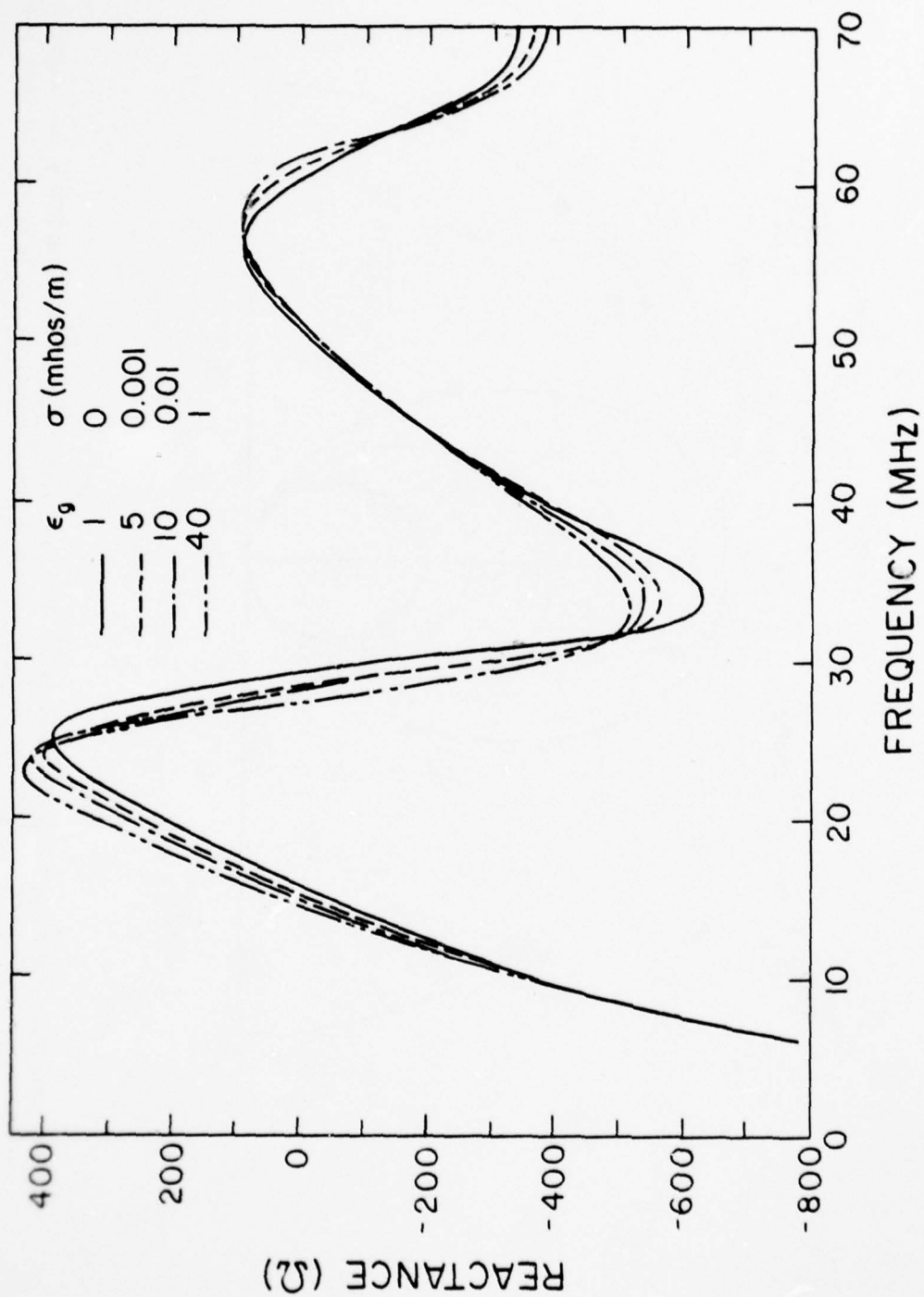


Figure 23. Input reactance of the antenna defined in Figure 22.

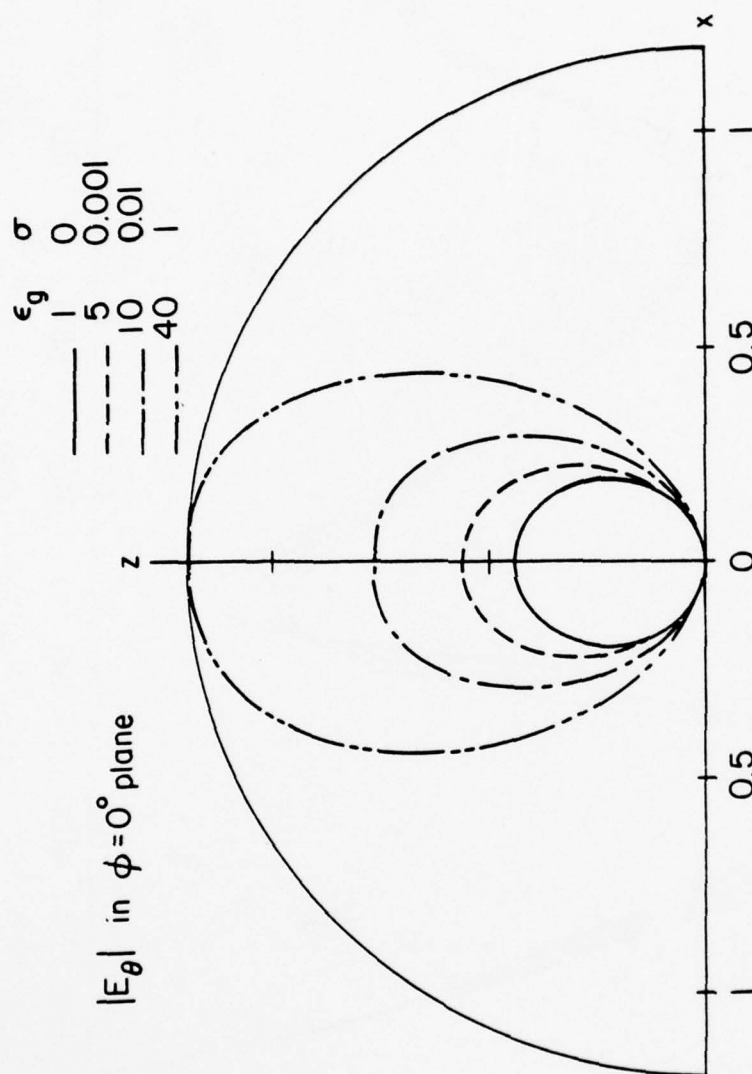


Figure 24. Far-field radiation pattern for the horizontal antenna defined in Figure 22 at 15 MHz. Note that the patterns are computed at $k_1 r = 500$, and in this plane, $|E_\phi|$ is negligible.

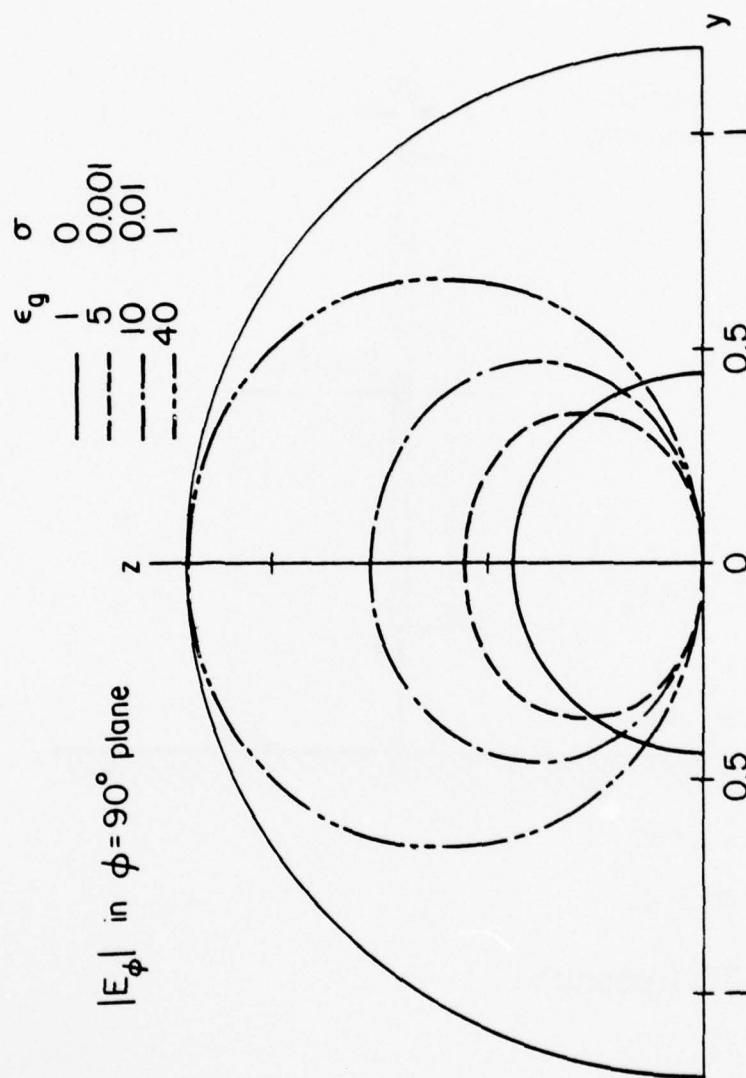


Figure 25. Far-field radiation pattern for the horizontal antenna defined in Figure 22 at 15 MHz. Note that the patterns are computed at $k_1 r = 500$, and in this plane, $|E_\theta|$ is negligible.

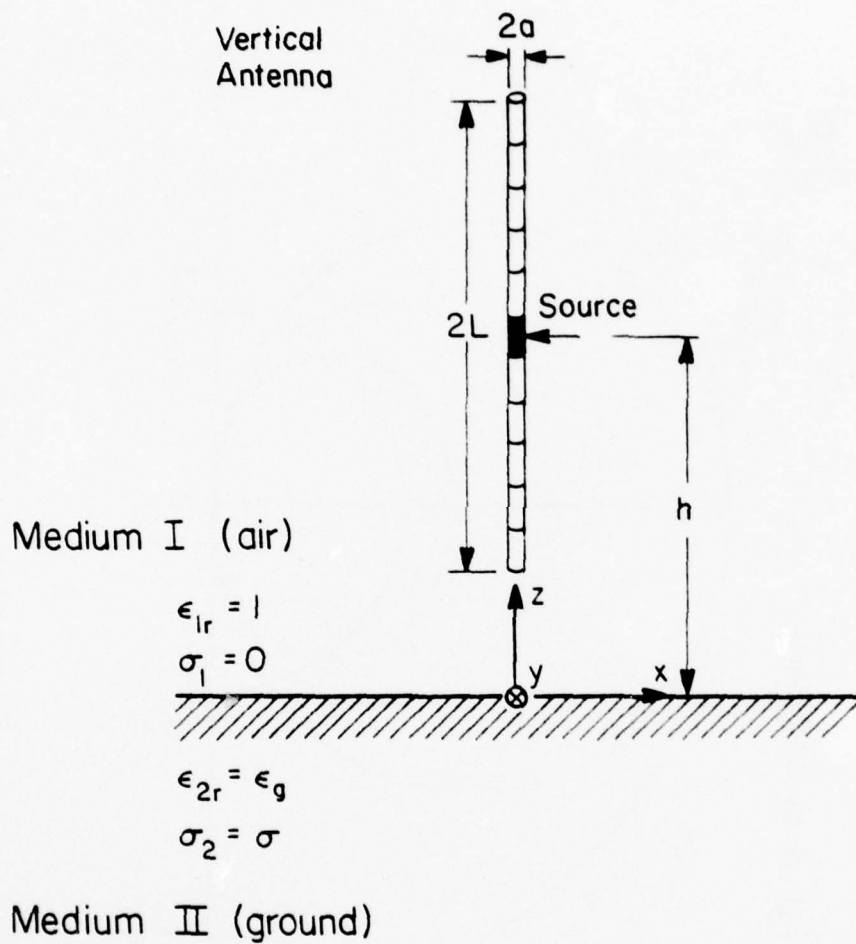


Figure 26. Center-fed vertical dipole over a lossy half-space.

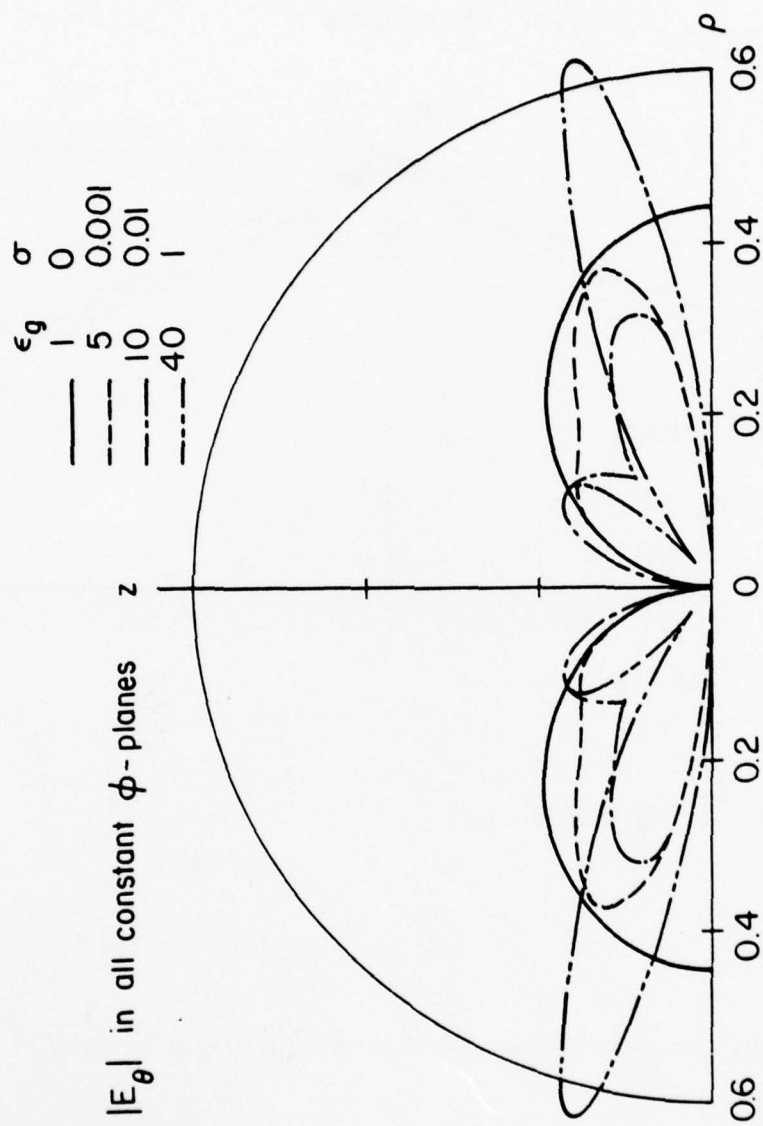


Figure 27. The far-field radiation pattern for a center-fed vertical dipole ($2L = 10m$, $h = 8m$, and $2a = 0.1m$) at 15 MHz. Note that the patterns are computed at $k_1 r = 500$, and in this example, $|E_\phi|$ is negligible.

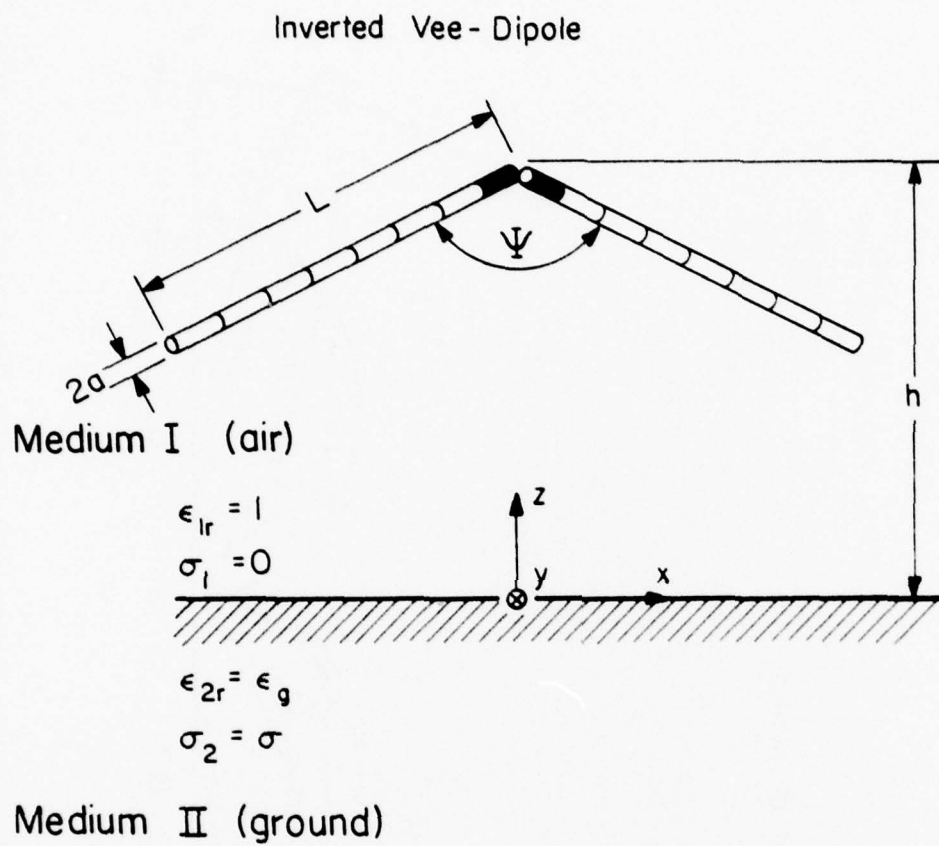


Figure 28. Center-fed inverted Vee-dipole over a lossy half-space.

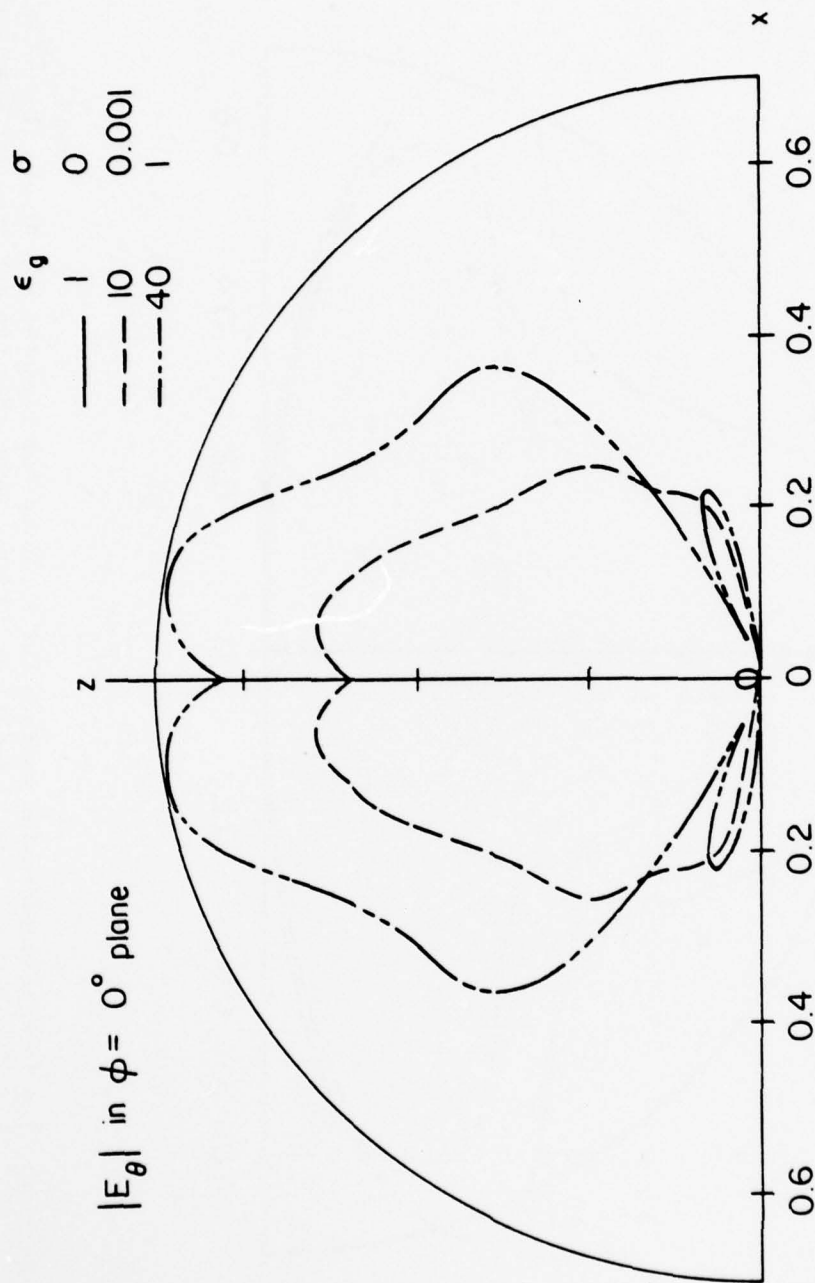


Figure 29. The far-field radiation pattern for a center-fed inverted Vee-dipole ($L = 7.5\text{m}$, $h = 10\text{m}$, $\psi = 90^\circ$, and $2a = 0.1\text{m}$) at 10 MHz. Note that the patterns are computed at $k_1 r = 500$, and in this plane, $|E_\phi|$ is negligible.

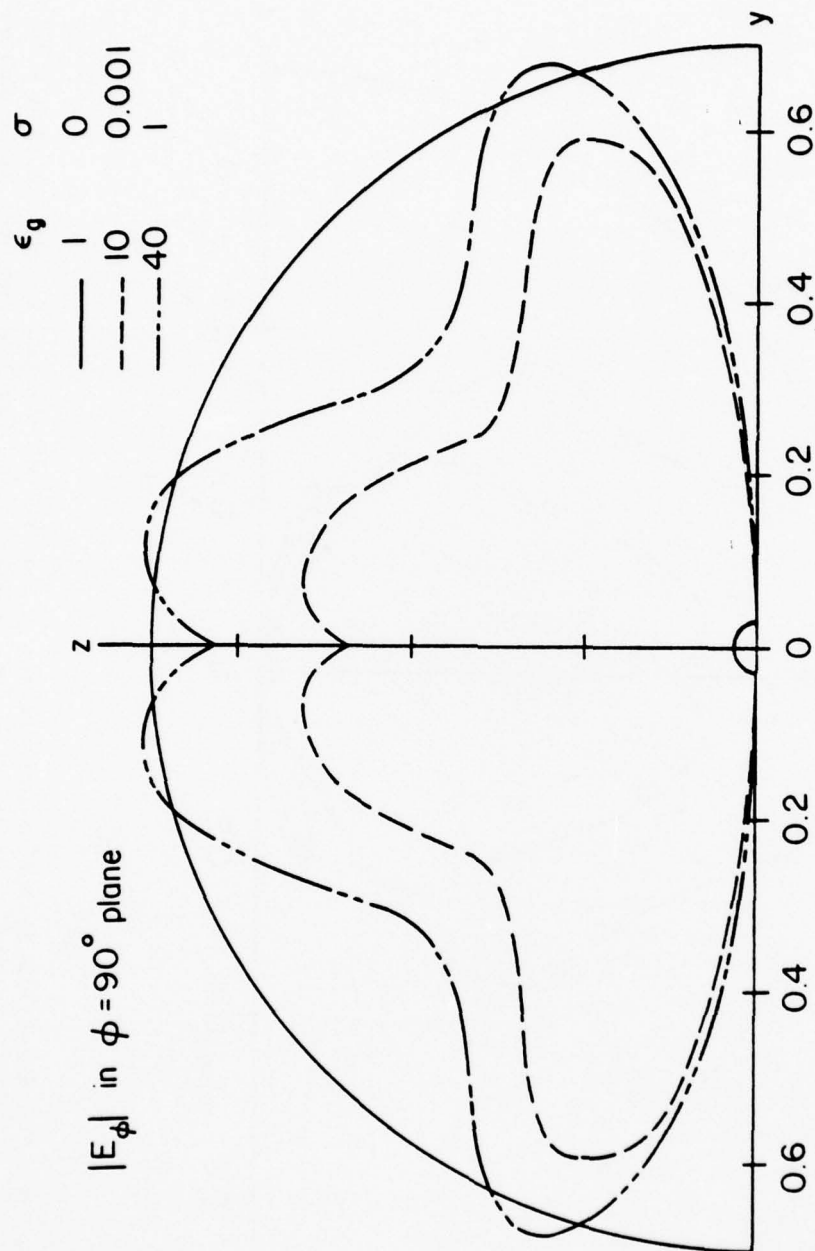


Figure 30. The far-field radiation pattern for a center-fed inverted Vee-dipole ($L = 7.5\text{m}$, $h = 10\text{m}$, $\psi = 90^\circ$, and $2a = 0.1\text{m}$) at 10 MHz. Note that the patterns are computed at $k_L r = 500$, and in this plane, $|E_\theta|$ is negligible.

6. CONCLUSIONS

Based on the steepest descent path (SDP) integration technique, an efficient numerical integration procedure is developed in Chapter 3 for computing the Sommerfeld infinite integrals present in the vector potential expressions of a current element radiating over a lossy half-space. Even though this procedure is about an order of magnitude faster than the latest reported Sommerfeld integration techniques, the computation time for a typical antenna problem can still become prohibitive. The reflection coefficient method (RCM) approximations, which are simply the first term in the asymptotic expansion of the Sommerfeld integrals, offer a simple closed-form solution valid only at the high end of the frequency spectrum and which cannot be employed in many practical situations. Also, the addition of the second term in the aforementioned asymptotic expansion to the RCM approximations is ruled out, since the resulting vertical vector potential components diverge from their respective exact integration values.

Chapter 4 presents a novel approach in which the transform domain representation of the vector potentials is approximated such that the resulting space-domain expressions do not require any kind of infinite integration. This approach has the merit of being computationally over an order of magnitude faster than the SDP technique of Chapter 3, while being accurate over a wide range of parameters of practical interest and, in addition, offers a simple and numerically manageable procedure for obtaining the near E- and H-field components.

The general computer program, listed in Appendix IV, is developed by employing the approximate formulas of Chapter 4 and is used to solve several

antenna geometries. With minor modifications, this program can be adapted to analyze most three-dimensional thin-wire antenna structures over a lossy half-space.

APPENDIX I

EVALUATION OF 0^{Π}_{h1x} , 0^{Π}_{h1z} , AND 0^{Π}_{v1z} AT $\theta_2 = 0$

In this appendix, the behavior of Equations (2.47) - (2.49) is studied at $\theta_2 = 0$. In their present forms, these integrals are not defined at $\rho_2 = 0$, although it is clear that their equivalent forms in (2.42) - (2.44) are bounded. Equation (2.47) can be expressed here for convenience as

$$0^{\Pi}_{v1z} = \frac{I_{v0}}{4\pi j} \int_{-\infty}^{\infty} \frac{\kappa \lambda}{\kappa \sqrt{k_1^2 - \lambda^2} + \sqrt{\kappa k_1^2 - \lambda^2}} H_0^{(2)}(\rho_2 \lambda) e^{-jz_2 \sqrt{k_1^2 - \lambda^2}} d\lambda \quad (I.1)$$

where $\rho_2 = r_2 \sin \theta_2$. The Hankel function in (I.1) is not bounded at $\theta_2 = 0$. To circumvent this difficulty, one replaces $H_0^{(2)}$ with its expansion from [22]

$$H_0^{(2)}(\rho_2 \lambda) = J_0(\rho_2 \lambda) - j \left[\frac{2}{\pi} \text{Ln} \frac{\gamma \rho_2 \lambda}{2} J_0(\rho_2 \lambda) + \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(m!)^2} \left(\frac{\rho_2 \lambda}{2} \right)^{2m} \phi(m) \right] \quad (I.2)$$

where γ is Euler's constant and $\phi(m)$ represents the harmonic series, i.e.,

$$\phi(m) = 1 + 1/2 + 1/3 + \dots + 1/m \quad (I.3)$$

Note that both J_0 and the summation terms in (I.2) are even functions of λ , hence, their contributions to the integral (I.1) are zero.

Substituting (I.2) into (I.1), one finally arrives at

$$0^{\Pi}_{v1z} = \frac{I_{v0}}{4\pi j} \int_{-\infty}^{\infty} \frac{\kappa \lambda}{\kappa \sqrt{k_1^2 - \lambda^2} + \sqrt{\kappa k_1^2 - \lambda^2}} \frac{-2j}{\pi} J_0(r_2 \sin \theta_2 \lambda) \cdot \text{Ln}(r_2 \lambda) e^{-jz_2 \sqrt{k_1^2 - \lambda^2}} d\lambda \quad (I.4)$$

which is obviously bounded at $\theta_2 = 0$. Introducing the change of variable $\lambda = k_1 \sin \xi$ into (I.4) and setting $\theta_2 = 0$, one finds

$$0^{\Pi}_{vlz} = \frac{I_{v0} k_1}{4\pi j} \int_{\Gamma} \frac{\kappa \sin \xi \cos \xi}{\kappa \cos \xi + \sqrt{\kappa^2 - \sin^2 \xi}} \frac{-2j}{\pi} \text{Ln}(k_1 r_2 \sin \xi) e^{-jk_1 r_2 \cos \xi} d\xi ;$$

$$\theta_2 = 0 \quad (I.5)$$

where path Γ is shown in Figure 2. In a similar fashion, one may obtain equivalent expressions for the remaining vector potential components at $\theta_2 = 0$, namely,

$$0^{\Pi}_{hlx} = \frac{I_{v0} k_1}{4\pi j} \int_{\Gamma} \frac{\sin \xi \cos \xi}{\cos \xi + \sqrt{\kappa^2 - \sin^2 \xi}} \frac{-2j}{\pi} \text{Ln}(k_1 r_2 \sin \xi) e^{-jk_1 r_2 \cos \xi} d\xi ;$$

$$\theta_2 = 0 \quad (I.6)$$

and

$$0^{\Pi}_{vlz} = 0 \quad ; \quad \theta_2 = 0 \quad . \quad (I.7)$$

APPENDIX II

ASYMPTOTIC EVALUATION

In this appendix, a general formulation is developed for a higher-order asymptotic evaluation of an integral with the following format:

$$u = \frac{1}{4\pi j} \int_{\Gamma} P(\xi) e^{-jkr \cos(\xi-\theta)} d\xi, \quad (\text{II.1})$$

where it is assumed that kr is a large parameter, $-\pi/2 < \theta < \pi/2$, $P(\xi)$ is a slowly varying function and path Γ is shown in Fig. 2. For large values of kr , one is usually interested in determining the asymptotic expression of (II.1); this is done by employing the method of the steepest-descent path integration. At the saddle point $\xi = \theta$, one can deform the integration path Γ to the steepest descent path (SDP) defined by $\text{Re}[\cos(\xi - \theta)] = 1$. Assuming that in this deformation no poles or branch points are encountered, one may express (II.1) as

$$u = \frac{1}{4\pi j} \int_{\text{SDP}} P(\xi) e^{-jkr \cos(\xi-\theta)} d\xi. \quad (\text{II.2})$$

Since on the SDP the relation $\text{Re}[\cos(\xi - \theta)] = 1$ holds, one can introduce the change of variable

$$\cos(\xi - \theta_2) = 1 - jt^2, \quad (\text{II.3})$$

or equivalently,

$$t = \sqrt{2} e^{-j\pi/4} \sin \frac{\xi - \theta}{2}, \quad (\text{II.4})$$

in which t is a real variable taking the domain $[-\infty, \infty]$. Substituting (II.4) into (II.2), one arrives at

$$u = \frac{e^{-jkr-j\pi/4}}{2\sqrt{2}\pi} \int_{-\infty}^{\infty} Q(t) e^{-krt^2} dt \quad (\text{II.5})$$

where

$$Q(t) = P(\xi) \sec \frac{\xi - \theta}{2}, \quad (\text{II.6a})$$

in which ξ is replaced with

$$\xi = \pm \left[\frac{\pi}{2} + j \ln(t^2 + j + |t| \sqrt{t^2 + 2j}) \right] + \theta, \quad t \lesseqgtr 0 \quad (\text{II.6b})$$

and \ln is interpreted as being its principal value. The complete asymptotic expansion procedure [23] is now used for the asymptotic evaluation of (II.5).

In this procedure, one first expands $Q(t)$ in a Taylor series

$$Q(t) = \sum_{n=0}^{\infty} \frac{Q^{(n)}(0)}{\Gamma(n+1)} t^n \quad (\text{II.7})$$

where $Q^{(n)}(0) = \frac{\partial^n}{\partial t^n} Q(t) \Big|_{t=0}$ and Γ is the Gamma function. Then (II.7) is

substituted into (II.5) to finally result in

$$u = \frac{e^{-jkr-j\pi/4}}{2\sqrt{2}\pi} \sum_{n=0}^{\infty} \frac{2^{-2n}}{n!} (kr)^{-n-1/2} Q^{(2n)}(0). \quad (\text{II.8})$$

In constructing the preceding equation, the following identity was used, viz.,

$$\int_{-\infty}^{\infty} t^n e^{-krt^2} dt = \begin{cases} (kr)^{-(1+n)/2} \Gamma[(1+n)/2] & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases} \quad (\text{II.9})$$

The task is now to determine $Q^{(2n)}$'s in terms of P . This is achieved by differentiating (II.6a) and arriving at

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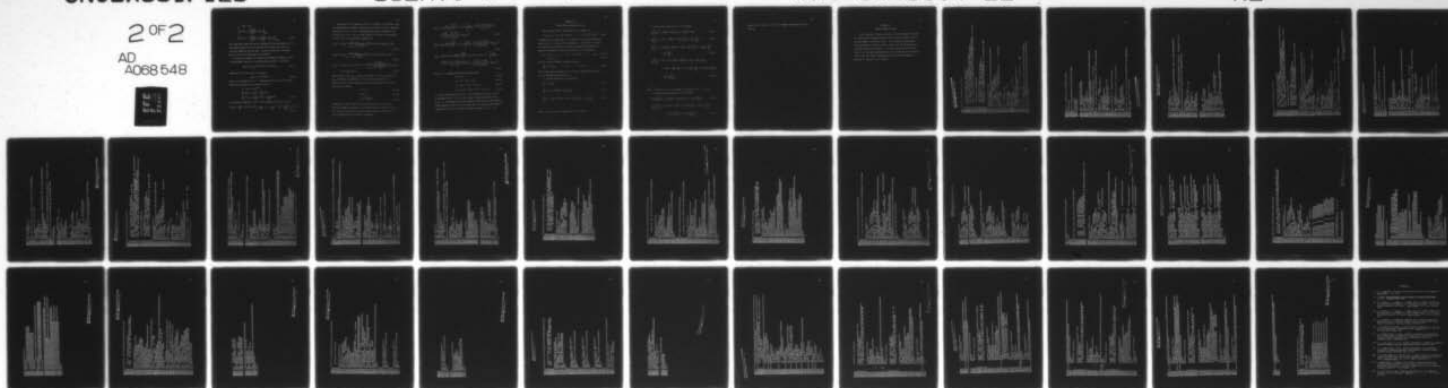
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$$\begin{cases} Q(0) = P(\theta) \\ Q^{(2)}(0) = 2j \left[\frac{d^2 P(\xi)}{d\xi^2} + \frac{1}{4} P(\xi) \right]_{\xi=\theta} \\ Q^{(4)}(0) = -4 \left[\frac{d^4 P(\xi)}{d\xi^4} + \frac{5}{2} \frac{d^2 P(\xi)}{d\xi^2} + \frac{9}{16} P(\xi) \right]_{\xi=\theta} \end{cases} \quad (II.10)$$

The higher-order terms can also be determined in the same fashion. It is worth emphasizing here that in deriving the preceding equations, the following assumption has been made: neither the poles nor the branch points of $P(\xi)$ are intercepted by the path SDP.

To present an example, the higher-order asymptotic expansion of the following Hankel function of the second kind and order ν is derived:

$$H_\nu^{(2)}(\Omega) = \frac{1}{\pi} \int_{\Gamma} e^{-j\nu\xi + j\nu\pi/2} e^{-j\Omega \cos\xi} d\xi \quad (II.11)$$

Comparing (II.11) with (II.1), one obtains

$$P(\xi) = 4j e^{-j\nu\xi + j\nu\pi/2} \quad (II.12)$$

where it is assumed that $\Omega \gg \nu$. Substituting (II.12) into (II.10) and simplifying the result, one finally arrives at

$$\begin{cases} Q(0) = 4j e^{j\nu\pi/2} \\ Q^{(2)}(0) = -8 \left[-\nu^2 + \frac{1}{4} \right] e^{j\nu\pi/2} \\ Q^{(4)}(0) = -16j \left[\nu^4 - \frac{5}{2} \nu^2 + \frac{9}{16} \right] e^{j\nu\pi/2} \end{cases} \quad (II.13)$$

The asymptotic expansion of $H_\nu^{(2)}$ is then determined using (II.8) to be

$$H_\nu^{(2)}(\Omega) = \sqrt{\frac{2}{\pi\Omega}} e^{-j(\Omega - \nu\pi/2 - \pi/4)} \left[1 + \frac{j}{2\Omega} \left(\frac{1}{4} - \nu^2 \right) - \frac{1}{8\Omega^2} \left(\nu^4 - \frac{5}{2} \nu^2 + \frac{9}{16} \right) \right] + O(\Omega^{-7/2}). \quad (II.14)$$

The purpose of this appendix has been to formulate the necessary steps for determining the asymptotic expansion of the vector potential components expressed in (2.47) - (2.49). Rearranging these three expressions into the form of Eq. (II.1), one can easily define a P function corresponding to each of the vector potential components as

$$P_{vlz}(\xi) = I_{v0} k_1 \kappa \frac{\sin \xi \cos \xi}{\kappa \cos \xi + \sqrt{\kappa - \sin^2 \xi}} H_0^{(2)}(k_1 \rho_2 \sin \xi) \exp(jk_1 \rho_2 \sin \xi) \quad (II.15)$$

$$P_{hlx}(\xi) = I_{h0} k_1 \frac{\sin \xi \cos \xi}{\cos \xi + \sqrt{\kappa - \sin^2 \xi}} H_0^{(2)}(k_1 \rho_2 \sin \xi) \exp(jk_1 \rho_2 \sin \xi) \quad (II.16)$$

$$P_{hlz}(\xi) = -j I_{h0} k_1 \cos \phi_2 \sin^2 \xi \cos \xi \frac{\cos \xi - \sqrt{\kappa - \sin^2 \xi}}{\kappa \cos \xi + \sqrt{\kappa - \sin^2 \xi}} H_1^{(2)}(k_1 \rho_2 \sin \xi) \cdot \exp(jk_1 \rho_2 \sin \xi) \quad (II.17)$$

where functions P_{vlz} , P_{hlx} , and P_{hlz} correspond to the 0^{Π}_{vlz} , 0^{Π}_{hlx} , and 0^{Π}_{hlz} expressions, respectively. Since the final results take a complicated form, the following notations are introduced for the ease of representation, namely,

$$c = \cos \theta_2 \quad (II.18a)$$

$$s = \sin \theta_2 \quad (II.18b)$$

$$q = \sqrt{\kappa - \sin^2 \theta_2} \quad (II.18c)$$

Performing the rather tedious differentiation needed in (II.10) and using the asymptotic expansion for the Hankel functions derived in (II.14), one finally arrives at the following two-term asymptotic expression for the aforementioned vector potential components:

$$\begin{aligned}
0^{\Pi}_{v1z} = I_{v0} \frac{2\kappa c}{\kappa c + q} \frac{e^{-jk_1 r_2}}{4\pi r_2} + I_{v0} j\kappa c \left[\frac{-2}{\kappa c + q} + \frac{A_1(3c - 2/c) + A_2 + \kappa c}{(\kappa c + q)^2} \right. \\
\left. + \frac{2s^2 A_1^2}{(\kappa c + q)^3} \right] \frac{e^{-jk_1 r_2}}{4\pi k_1 r_2^2} + O(k_1 r_2)^{-3}
\end{aligned} \quad (II.19)$$

$$\begin{aligned}
0^{\Pi}_{h1x} = I_{h0} \frac{2c}{c + q} \frac{e^{-jk_1 r_2}}{4\pi r_2} + I_{h0} jc \left[\frac{A_3}{c + q} + \frac{A_2 + c}{(c + q)^2} \right] \frac{e^{-jk_1 r_2}}{4\pi k_1 r_2^2} + O(k_1 r_2)^{-3}
\end{aligned} \quad (II.20)$$

$$\begin{aligned}
0^{\Pi}_{h1z} = I_{h0} 2\cos \phi_2 \sec \frac{c - q}{\kappa c + q} \frac{e^{-jk_1 r_2}}{4\pi r_2} + I_{h0} \cos \phi_2 jsc(c - q) \left[\frac{A_4}{\kappa c + q} \right. \\
\left. + \frac{A_1(5c - 2/c + 2s^2/q) + A_2 + \kappa c}{(\kappa c + q)^2} + \frac{2s^2 A_1^2}{(\kappa c + q)^3} \right] \frac{e^{-jk_1 r_2}}{4\pi k_1 r_2^2} + O(k_1 r_2)^{-3}
\end{aligned} \quad (II.21)$$

where $A_1 - A_4$ expressions take the following form:

$$A_1 = \kappa q + c \quad (II.22a)$$

$$A_2 = (s^4 - 2\kappa s^2 + \kappa)/q^3 \quad (II.22b)$$

$$A_3 = 2s^2/q^2 + (3c - 2/c)/q - 2 \quad (II.22c)$$

$$A_4 = s^2 c/q^3 + s^2/q^2 + 2(3c - 1/c)/q - 6 \quad ; \quad (II.22d)$$

s, c , and q were defined in (II.18). It is interesting to point out that in deriving (II.19) - (II.21), one initially encounters singular terms at $\theta_2 = 0$. As expected, after some algebraic manipulations, these singular terms completely cancel out in all three cases, and the two-term asymptotic expressions for the vector potentials remain bounded for $0 \leq \theta_2 < 90$.

APPENDIX III

VARIOUS PARTIAL DERIVATIVES OF g

The free-space Green's function g , which is defined as

$$g = g(x_1, x_2, x_3) = \exp(-jkr)/4\pi r \quad ; \quad r = [x_1^2 + x_2^2 + x_3^2]^{1/2} \quad (\text{III.1})$$

is singular only at the point $r = 0$, therefore, its partial derivatives exist for all $r > 0$ and can be represented in a closed form. In order to express these derivatives in an organized manner and suitable for numerical evaluation, the auxiliary function R_i is introduced as

$$R_i = (-1)^i \{1 \times 2 \times 4 \times \dots \times (2i - 2) + jkr[1 \times 3 \times 5 \times \dots \times (2i - 3)]\} r^{-2i};$$

$$i = 1, 2, 3, \dots \quad (\text{III.2})$$

where R_i has the following convenient property

$$\frac{\partial}{\partial x_j} R_i = x_j R_{i+1} \quad ; \quad i = 1, 2, \dots \quad ; \quad j = 1, 2, 3 \quad . \quad (\text{III.3})$$

The various partial derivatives of g can now be expressed in terms of x_j , R_i , and lower-order partials of g .

a) Partial derivatives of one variable

$$\frac{\partial}{\partial x_j} g = x_j R_1 g \quad (\text{III.4})$$

$$\frac{\partial^2}{\partial x_j^2} g = (R_1 + x_j^2 R_2) g + x_j R_1 \frac{\partial}{\partial x_j} g \quad (\text{III.5})$$

$$\frac{\partial^3}{\partial x_j^3} g = (3x_j R_2 + x_j^3 R_3) g + 2(R_1 + x_j^2 R_2) \frac{\partial}{\partial x_j} g + x_j R_1 \frac{\partial^2}{\partial x_j^2} g \quad (\text{III.6})$$

...

...

where $j = 1, 2, 3$ holds for Equations (III.4) - (III-6).

b) Mixed partial derivatives of two variables

$$\frac{\partial^2}{\partial x_j \partial x_k} g = x_j x_k R_2 g + x_k R_1 \frac{\partial}{\partial x_j} g = x_j x_k (R_1^2 + R_2) g \quad (\text{III.7})$$

$$\frac{\partial^3}{\partial x_j^2 \partial x_k} g = x_k (R_2 + x_j^2 R_3) g + 2x_j x_k R_2 \frac{\partial}{\partial x_j} g + x_k R_1 \frac{\partial^2}{\partial x_j^2} g \quad (\text{III.8})$$

$$\begin{aligned} \frac{\partial^4}{\partial x_j^3 \partial x_k} g &= x_k (3x_j R_3 + x_j^3 R_4) g + 3x_k (R_2 + x_j^2 R_3) \frac{\partial}{\partial x_j} g + 3x_j x_k R_2 \frac{\partial^2}{\partial x_j^2} g \\ &\quad + x_k R_1 \frac{\partial^3}{\partial x_j^3} g \end{aligned} \quad (\text{III.9})$$

$$\begin{aligned} \frac{\partial^4}{\partial x_j^2 \partial x_k^2} g &= [R_2 + (x_j^2 + x_k^2) R_3 + x_j^2 x_k^2 R_4] g + x_k (R_2 + x_j^2 R_3) \frac{\partial}{\partial x_k} g \\ &\quad + 2x_j (R_2 + x_k^2 R_3) \frac{\partial}{\partial x_j} g + (R_1 + x_k^2 R_2) \frac{\partial^2}{\partial x_j^2} g + 2x_j x_k R_2 \frac{\partial^2}{\partial x_j \partial x_k} g \\ &\quad + x_k R_1 \frac{\partial^3}{\partial x_j^2 \partial x_k} g \end{aligned} \quad (\text{III.10})$$

...

...

where $j \neq k$ and $j, k = 1, 2, 3$ is assumed for Equations (III.7) - (III.10).

c) Mixed partial derivatives of three variables

$$\frac{\partial^3}{\partial x_j \partial x_k \partial x_l} g = x_j x_k x_l R_3 g + 2x_k x_l R_2 \frac{\partial}{\partial x_j} g + x_l R_1 \frac{\partial^2}{\partial x_j \partial x_k} g \quad (\text{III.11})$$

$$\begin{aligned} \frac{\partial^4}{\partial x_j^2 \partial x_k \partial x_l} g &= x_k x_l (R_3 + x_j^2 R_4) g + 3x_j x_k x_l R_3 \frac{\partial}{\partial x_j} g + 2x_k x_l R_2 \frac{\partial^2}{\partial x_j^2} g \\ &\quad + x_j x_l R_2 \frac{\partial^2}{\partial x_j \partial x_k} g + x_l R_1 \frac{\partial^3}{\partial x_j^2 \partial x_k} g \end{aligned} \quad (\text{III.12})$$

...

...

where $j \neq k \neq l$ and $j, k, l = 1, 2, 3$ is assumed for Equations (III.11) - (III.12).

APPENDIX IV
COMPLETE COMPUTER LISTING

In this appendix, a complete listing of a Fortran program is listed for analyzing wire antenna structures over a lossy half-space, based on the developments of Chapters 4 and 5. Also included are subroutines for evaluating the correction vector potential via the SDP integration technique introduced in Chapter 3. To avoid confusion, care has been taken to use the same symbols and names in the program as were introduced in the text, and comment statements have been included frequently to describe the function of each routine.


```

025000C***** COMPUTE THE IMPEDANCE MATRIX
026000C
027000C DEL=ANTL/N
028000C
029000C***** USE FINITE-DIFFERENCE TECHNIQUE FOR EINC
030000C
031000C DO 100 I=1,N
032000C   X=(I-1)*DEL
033000C   Y=X+DEL/2.
034000C
035000C CALL SI(DEL,RADIUS,X,ZI(I))
036000C CALL SI(DEL,RADIUS,X,ZM(I))
037000C   CONTINUE
038000C ZL(I)=ZM(I)
039000C DO 100 J=1,N
040000C   ZL(I)=ZL(I)+ZL(J)+ZL(J)/(DEL/2.)*2+AK1**2+ZI(J)
041000C   ZTOT(I,J)=ZTOT(I,J)+DEL*CI0
042000C   CONTINUE
043000C IF(EPSH.EQ.1.0)AND(SIGMA.EQ.0.0) GO TO 200
044000C
045000C***** ADD THE GROUND CONTRIBUTIONS TO THE MATRIX ELEMENTS
046000C
047000C Z2=2*RH $ PHID=0.
048000C CI=(0.,0.) $ C2=DEL**2*CI0
049000C DO 100 I=1,N
050000C   RH02=(1-1)*DEL
051000C   RH02=RH02**2+Z2**2
052000C   TH=ACOS(Z2/RH02)
053000C   CALL SESAPP(UEX,UEY,UEZ,C1,C2)
054000C   ZTOT(I,I)=ZTOT(I,I)+UEX
055000C   CONTINUE
056000C
057000C***** USE SYMMETRY TO COMPLETE THE MATRIX
058000C
059000C DO 200
060000C   CONTINUE
061000C   FORMAT(/,4(" ",G10.4," ",",",G10.4,""))
062000C   DO 300 I=1,N
063000C     DO 300 J=1,N
064000C       IF(I.EQ.J) GO TO 510
065000C       ZTOT(I,J)=ZTOT(I,J-1)
066000C       ZTOT(J,I)=ZTOT(I,J)
067000C     CONTINUE
068000C
069000C***** INVERT THE MATRIX AND COMPUTE CURRENTS AND THE IMPEDANCE
070000C
071000C CALL ALINY(Z,N,ZI,ZTOT,UEI,IER)
072000C MU2=M/2+1

```

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```

045000 00 400 I=1,11
047000 JANT(1)=ZTOT(1,NU2)
048000 400 CONTINUE
049000 PRINT049
049500 049 FORMAT(///,"ANTENNA CURRENTS(AMPS), DUE TO 1-VOLT EXCITATION")
049500 PRINT050,(JANT(1),I=1,N)
050000 02IMP=1./JANT(NU2)
050700 PRINT060,02IMP
051700 060 FORMAT(///,"ANTENNA IMPEDANCE(OMMS)= (" ,G14.6," , "G14.6," )",//)
052700
053700 *** COMPUTE THE E-FIELD PATTERNS IN THE PHIPHIOP PLANE
054700
055700 AKIM05500
056700 K0=AKIM/AK1
057700 3131 RFAD0900,PHIOP
058200 IF(PHIOP.LT.0.) GO TO 1111
058700 PRINT970
060700 PHIP=PHIOP+PI/180.
061700 DO 700 I=1,45
062700 TH0P=2.*I
063700 TH0=TH0P+PI/1800
064700 UETH=UEPHI*(0.,0.)

065700
066700 00 750 K=1,N
067700 ZCURR=0 YCURR=0.
068700 XCURR=(K-(N+1)/2)*DEL
069700 UIZ=(0.,PI/2)
071700 UIZ=JANT(UIZ)
072700 CALL DEPAT(UIZ,U12,U1,U2)
073700 UETH=UETH+U1*DEL
074700 UEPHI=UEPHI+U2*DEL
075700 750 CONTINUE
076700 ETH=CABS(UETH) $ EPHI=CABS(UEPHI)
077700 PRINT975,AKIM,TH0P,PHIOP,ETH,EPHI
078700 700 CONTINUE
079700 975 FORMAT(///)
080700 975 PRINT075,AKIM= " ,G0.2," TH0= " ,G0.2," PHIO= " ,G0.2,
081700+ " MAG-ETH= " ,G14.6," MAG-EPHI= " ,G14.6)
082700 GO TO 3131
083700 END

```

```

001000 PROGRAM VERTIC(INPUT,OUTPUT)
002000
002100 CENTER-FED VERTICAL WIRE ANTENNA
002200 ANTENNA-LENGTH (METERS)
002300 RADIUS ANTENNA-RADIUS (METERS)
002400 N ANTENNA-HEIGHT ABOVE THE LOSSY HALF-SPACE (MEASURED FROM ANTENNA-CENTER)
002500 EPSM, SIGMA GROUND PARAMETERS, SIGMA IN MMUS/METERS
002600 N# NUMBER OF CURRENT PATCHES (OOD FOR A CENTER-FED ANTENNA)
002700 PHI#P THE PHI-PLANE IN WHICH PATTERN IS COMPUTED (AS A FUNCTION OF THETA)
003000
003100 *****INPUT*****
003200 READ FREQUENCY (MHZ)
003300 IF EPSM, SIGMA (DEFINING THE GROUND)
003400 IF EPSM, SIGMA THEN READ A NEW FREQUENCY
003500 READ PHI#P (DEGREES) / PATTERN-PLANE
003600 IF PHI#P, PATTERN NOT COMPUTED AND NEW GROUND IS READ
004000
004100 IMPLICIT COMPLEX(C,UI)
004200 REAL COS,CARS
004300 COMPLEX KAPPA,ZTOT(21,21),JANI(21),ZI(21),ZK(21),ZL(21)
004400 COMPLEX ZEXTRA(4)
004500 ---ZEXTRA MUST BE DIMENSIONED AT LIST 2*N+1 LONG
004600 COMMON/PI/ PI,EP5M,CJ
004700 COMMON/UBS/ H2,IM2,PHI2
004800 COMMON/PARAM/ A1,KAPPA,CIN
004900 COMMON/PATERN/ H0,TH0,PHI0
005000 COMMON/PATCH/ XCURR,YCURR,ZCURR
005100 PI=4.*3.141592653589793
005200 CJ=1.E-9/(50.*PI) S CJ=(0.,1.)
006000 READ 900 FREQ
006100 FORMAT(F14.5)
006200 IF (FREQ.EQ.0) STOP
006300 FREQ=FREQ*1.E6
006400 1111 CONTINUE
006500 READ 900 EPSM
006600 IF (EPSM.EQ.0) GO TO 2121
006700 READ 900 SIGMA
006800 PH1=900 FREQ,EP5M,SIGMA
006900 A1=2.*PI*FREQ*0.5*E6
007000 KAPPA=EPSM-CJ*SIGMA/(2.*PI*FREQ*0.5*E6)
007100 CIN=1./CJ*2.*PI*FREQ*0.5*E6
007200 KAPPA=1.7
007300 S HADLUS=0.5 S N=21
007400 PH1=1020 FREQ,KAPPA
007500 FORMAT(//,"FREQ=",G12.4,"",KAPPA=","G12.4,"",G14.6)
007600

```

```

002440 PRINT#30,AUTL,RADIUS,ZH*H
002450 930 FORMAT(//,"ANT-L= ",G10.5,"ANT-HAU= ",G12.6,"ANT-HEIGHT= ",
002460 G10.5," N= ",I4)
002500C
002510C ***** COMPUTE THE IMPEDANCE MATRIX
002600C
002610C DEL=AUTL/N
002620C
002630C ***** USE FINITE-DIFFERENCE TECHNIQUE FOR EINC
002640C
002650C DO 100 J=1,N
002660C K=(I-1)*DEL
002670C AR=K+DEL/2.
002680C CALL SI(DEL,RADIUS,X,Z1(I))
002690C 100 CONTINUE
002700C ZL(1)=2R(I)
002710C DO 210 J=2,N
002720C ZL(1)=2R(I-1)
002730C 210 Z1(J)=2R(J)+ZL(J)-2.*Z1(J)/(DEL/2.)*2+AK1**2+Z1(J)
002740C Z1(J)=Z1(J)-Z1(J)*DEL*CI0
002750C 100 CONTINUE
002760C DO 150 J=2,N
002770C Z1(J)=Z1(J)-Z1(J-1)
002780C 150 Z1(J)=Z1(J)-Z1(J-1)
002790C IF(EPSM.EQ.1.,AND,SIGMA.EQ.0.) GO TO 200
002800C ***** ADD THE GROUND CONTRIBUTIONS TO THE MATRIX ELEMENTS
002810C
002820C DO 310 J=1,N
002830C C2=(0.,0.)
002840C C1=-DEL**2*CI0
002850C PH12=0.
002860C DO 110 I=1,NPP
002870C K2=Z2-Z1-AR+I*DEL
002880C CALL SE$APP(UEX,UEY,DEL,C1,C2)
002890C ZEXTHA(I)=C2
002900C 110 CONTINUE
002910C DO 115 J=1,N
002920C DO 115 J=1,N
002930C Z1(J)=Z1(J)+ZEXTHA(I+J-1)
002940C 115 CONTINUE
002950C ***** USE SYMMETRY TO COMPLETE THE MATRIX
002960C
002970C DO 350 J=1,N
002980C 350 CONTINUE
002990C 950 FORMAT(/,4(" ",G10.4," ",G10.4,""))
003000C
003010C K=I-1

```

```

04000 DO 500 J=1,N
04100 Z10T(1,J)=Z10T(J,1)
04200 300 CONTINUE
04300 ***INVERT THE MATRIX AND COMPUTE CURRENTS AND THE IMPEDANCE
04350C
04390C CALL XINVZ(N,Z10T,UDEL,IER)
04400 NO2=N/2+1
04500 DO 400 I=1,N
04600 DO 400 J=1,N
04700 JANT(1)=Z10T(1,NO2)
04800 400 CONTINUE
04840 PRINT=49
04880 849 FORMAT(///,"ANTENNA CURRENTS(AMPS), DUE TO 1-VOLT EXCITATION")
04900 PRINT=50, JANT(1), 1=1,N)
05000 UZIMP=1/JANT(NO2)
05070 PRINT=60,UZIMP
05170 800 FORMAT(///,"ANTENNA IMPEDANCE(OHMS)= ('G14.6,' ",G14.6," )",//)
05270C
05370C ***COMPUTE THE E-FIELD PATTERNS IN THE PHI=PHI0P PLANE
05470C
05570 ANIR0=500
05670 K0=AKIR0/AK1

05770 5151 READ900,PHI0P
05870 PRINT=70,LI=0, GO TO 1111
05970 PHI0=PHI0P*PI/180.
06070 PHI0=PHI0P*PI/180.
06170 DO 700 I=1,45
06270 I=I+1
06370 I=I+1
06470 UETH=UEPHI/180
06570C
06670C
06770 DO 750 K=1,N
06870 XCURR=0. YCURR=0
06970 ZCURR=H*ANTL/2.+(N-.5)*DEL
07070 U1A=(0.,0.)
07170 U1Z=JANT(A)
07270 CALL SEPAR(U1A,U1Z,U1,U2)
07370 UETH=UETH+U1*DEL
07470 UEPH1=UEPH1+U2*DEL
07570 750 CONTINUE
07670 ETH=CABS(UETH) * EPH1=CABS(UEPH1)
07770 PRINT=75,AKIR0, I=1,N,PHI0P,ETH,EPH1
07870 700 CONTINUE
07970 970 FORMAT(///)
08070 975 FORMAT(" KIR0=",G0.2," I=1,N=",G0.2," PHI0=",G0.2,
08170 " MAG-E=TH=",G14.6," MAG-EPHI=",G14.6)
08270 GO TO 5151
08370 END

```


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001000 PROGRAM VEEDIP(INPUT,OUTPUT)
002000
003000 INVERTED VEE-DIPLOLE ANTENNA OVER GROUND
004000 ANTILEG= THE LENGTH OF EACH ANTENNA LEG (METERS)
005000 RADIUS= ANTENNA-RADIUS (METERS)
006000 H= ANTENNA-HEIGHT ABOVE THE LOSSY HALF-SPACE (MEASURED FROM ANTENNA-CENTER)
007000 EPSM, SIGMA= GROUND PARAMETERS (SIGMA IN MHOS/METERS)
008000 N= NUMBER OF CURRENT PATCHES (EVEN FOR A CENTER-FED ANTENNA)
009000 PHI0= THE PHI-PLANE IN WHICH PATTERN IS COMPUTED (AS A FUNCTION OF THETA)
010000
011000 *****INPUT*****
012000
013000 READ FREQUENCY (MHZ)
014000 READ EPSM, SIGMA (DEFINING THE GROUND)
015000 IF EPSM<0 THEN READ A NEW FREQUENCY
016000 READ PHI0 (DEGREES), PATTERN-PLANE
017000 IF PHI0<0, PATTERN NOT COMPUTED AND NEW GROUND IS READ
018000
019000 IMPLICIT COMPLEX(C,U)
020000
021000 REAL COS, CABS
022000 COMPLEX KAPPA, ZI(22), ZR(22), ZL(22)
023000 COMMON/PL/ F1, EPS0, CJ
024000 COMMON/UNS/ K2, TH2, PHI2
025000 COMMON/PARAM/ AK1, KAPPA, C10
026000 COMMON/PATTEM/ K0, TH0, PHI0
027000 COMMON/PATCH/ ACUR, YCURN, ZCURN
028000 PLS4=ATAN(1.) $ ACUR=EPS0=1.E-9/(36.*PI) $ CJ=(0.,1.)
029000 PLS4=ATAN(1.) $ ACUR=EPS0=1.E-9/(36.*PI) $ CJ=(0.,1.)
030000 2121 CONTINUE
031000
032000 READ YCURN, PLS4
033000 YCURN= PLS4*(F14.0)
034000
035000 IF (PPLU, L1) $ STOP
036000 PPLU=PPLU+1.E6
037000 1111 CONTINUE
038000 READ YCURN, EPSM
039000 IF (EPSM, L1) $ GO TO 2121
040000 READ YCURN, SIGMA
041000 READ YCURN, ANTIS1
042000 PPLU=YCURN, PPLU, EPSM, SIGMA
043000 ANTIS1=APL*PPLU/3.E6
044000 KAPPA=EPSM-CJ=SIGMA/(2.*PI)*PPLU*EPSM
045000 C1=1./(CJ+2.*APL*PPLU*EPSM)
046000
047000 ANTILEG=7.5 $ H=10. $ RADIUS=.05 $ N=22
048000
049000 PHI=180., PPLU, KAPPA
050000

```



```

05100  PRINT  FORMAT(//,"RADIUS="G10.5,"KAPPA="G12.4,"",G14.0)
05200  PRINT 50,ANTISI,RADIUS
05300  PRINT 50,ANTISNA-LEG,610.5,"INVERTED-VEE ANGLE="G10.5,
05400  "ANTI-RADIUS="G10.5,"ANTI-HEIGHT="G10.5)
05425  PRINT 55,
05430  PRINT 55  FORMAT(//,"N="G15)
05450  ANTISI=ANTISI*PI/180.
05500C ***** COMPUTE THE IMPEDANCE MATRIX
05700C
05800C  N02=N/2
05900C  DEL=ARL*LEG/N02
06000C
06100C ***** USE FINITE-DIFFERENCE TECHNIQUE FOR EINC(SYMMETRY IS USED)
06200C
06300C  DO 100 I=1,N02
06400C  X=(1-I)*DEL
06500C  A=X+DEL/2.
06600C  CALL SI(DEL,RADIUS,X,ZI(I))
06700C  CALL SI(DEL,RADIUS,X,ZI(I))
06800C  100 CONTINUE
06900C  ZI(I)=ZK(I)
07000C  DO 110 I=2,N02
07100C  110 ZI(I)=ZK(I-1)
07200C
07300C  DO 120 J=1,N02
07400C  ZIOT(1,J)=ZK(J)+ZI(J)-2.*ZI(J)/(DEL/2.)*2.*AK1*2.*ZI(J)
07500C  120 CONTINUE
07600C  DO 130 I=2,N02
07700C  130 ZIOT(1,J)=ZIOT(1,N02,J+N02)=ZIOT(1-1,J-1)
07800C
07900C *****
08000C  DO 140 I=1,N02
08100C  A=XN02+1-I
08200C  DO 150 J=1,N02
08300C  X=(N02-1+J)*DEL+(J-5)*DEL*GOS(P1-ANTISI)+2.*RADIUS*SIN(P1-ANTISI)
08400C  A=XN02+J*(J-5)*DEL*GOS(P1-ANTISI)-2.*RADIUS*GOS(P1-ANTISI)+RADIUS)
08500C  CALL SI(DEL,RN02,X,ZI(I))
08600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
08700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
08800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
08900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
09900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
10900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
11900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
12900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
13900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
14900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
15900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
16900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
17900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
18900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19100C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19200C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19300C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19400C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19500C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19600C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19700C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19800C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
19900C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))
20000C  CALL SI(DEL,RN02,X+DEL/2.,Z1(KKX))

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06000C*** ADD THE GROUND CONTRIBUTIONS TO THE MATRIX ELEMENTS(SUMMETRY IS USED)
08100C
08200C
08250 DU 210 I=1,N02
08270 DU 210 J=1,N02
08280 K02=ABS(FLOAT(I-J))*DEL*SIN(ANISI/2)
08290 Z2=2*(M-(N02-I+.5)*DEL*COS(ANISI/2))+ABS(FLOAT(I-J))*DEL*COS(ANISI/2)
08300 PH12=0
08310 K2=SUMF(K02**2+Z2**2)
08320 I=2,ATAJ(K02/22)
08330 C1=DEL**2*C1+COS(ANISI/2)
08340 C2=DEL**2*C2+SIN(ANISI/2)
08350 CALL SESAPP(UEX,UEY,UEZ,C1,C2)
08360 UET=UEX*SIN(ANISI/2)+UEZ*COS(ANISI/2)
08370 Z101(I,J)=Z101(I,J)+UET
08380 Z101(N+1-J,N+1-I)=Z101(I,J)+UET
08390 C10 CONTINUE
08400 DU 220 I=1,N02
08410 DU 220 J=1,N02
08420 IF (J.NE.1.AND.J.NE.1) GO TO 220
08430 K02=(N02-I+J)*DEL*SIN(ANISI/2)
08440 Z2=2*(M-(N02-I+J)*DEL*COS(ANISI/2))
08450 PH12=
08460 K2=SUMF(K02**2+Z2**2)
08470 I=2,ATAJ(K02/22)
08480 CALL SESAPP(UEX,UEY,UEZ,C1,C2)
08490 UET=UEX*SIN(ANISI/2)+UEZ*COS(ANISI/2)
08500 Z101(I,J+N02)=Z101(I,J+N02)+UET
08510 C20 CONTINUE
08520 DU 230 I=2,N02
08530 K2=K02+2
08540 DU 230 J=K,N
08550 Z101(I,J)=Z101(I-1,J-1)
08560 C20 CONTINUE
08570C***USE SYMMETRY TO COMPLETE THE MATRIX
08580C
08590C CONTINUE
08600 C50 FORMAT(/,4(" ",G10.4," ",G10.4,""))
08610 DU 500 I=2,N
08620 K=1-)
08630 DU 500 J=1,N
08640 Z101(I,J)=Z101(J,I)
08650 C50 CONTINUE
08660C***INVERT THE MATRIX AND COMPUTE CURRENTS AND THE IMPEDANCE
08670C
08680C CALL XINVZ(M,22,Z101,UET,IEM)
08690 DU 400 I=1,N
08700 JANT(I)=Z101(I,N/2)+Z101(I,N/2+1)

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11100 400 CONTINUE
11100 PH1=1049
11100 649 FORMAT(///,"ANTENNA CURRENTS(AMPS) DUE TO 1-VOLT EXCITATION
11100 649 APPLIED TO EACH OF THE TWO CENTER PATCHES")
11350+ PRINTN50, JANT(1), I=1,N
11400 PRINTN50, JANT(N/2)
11500 UZ1P=2, UZIMP
11600 PH1=1049, UZIMP
11700 860 FORMAT(///,"ANTENNA IMPEDANCE(OHMS)= (" ,G14.6," " ,G14.6," " ),//)
11800C
11900C**COMPUTE THE E-FIELD PATTERNS IN THE PH1=PHIMP PLANE
12000C
12100 AN1N=500
12200 K=AKIN0/AN1
12300 5131 READ900, PHIMP
12400 1P1PHIMP.LT.0.) GO TO 1111
12500 PRINT970
12600 PH1=PHIMP*PI/180.
12700 DO 700 I=1,46
12800 TH0P=2*(I-1)
12900 TH0P=TH0P*PI/180
13000 UETH=UEPHI*(0.,0.)
13100C
13200C
13200 DO 750 K=1,NU2
13220 YCURR=0.
13240 ZCURR=0.
13260 XCURR=0.
13280 XCURR=-(NU2-I+.5)*DEL*COS(ANTSI/2.)
13300 UIX=JANT(K)*SIN(ANTSI/2.)
13320 UIZ=JANT(K)*COS(ANTSI/2.)
13340 CALL SEPAI(UIX,UIZ,UI,N2)
13360 UETH=UETH+UI*DEL
13380 UEPHI=UEPHI+U2*DEL
13400 XCURR=XCURR
13420 UIX=JANT(N-N+1)*SIN(ANTSI/2.)
13440 UIZ=JANT(N-N+1)*COS(ANTSI/2.)
13460 CALL SEPAI(UIX,UIZ,UI,U2)
13480 UETH=UETH+UI*DEL
13500 UEPHI=UEPHI+U2*DEL
13520 750 CONTINUE
13540 ETH=CABS(UETH)
13560 PH1=975,AKIN0,TH0P,PHIMP,ETH,EPHI)
13580 700 CONTINUE
13600 970 FORMAT(///)
13620 970 FORMAI(" K1N0=" ,G0.2," TH0=" ,G0.2," PH1=" ,G0.2,
13640 970 MAG-ETH=" ,G14.6," MAG-EPHI=" ,G14.6)
13660 60 TO 5131
13680 END

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COMPUTE THE PARAMETER FIELD APPROXIMATION FOR E-THETA AND E-PHI.
 THE OBSERVATION POINT IS LOCATED AT (R0,TH0,PHI0)
 AND THE CURRENT ELEMENT IS LOCATED AT (RCURR,YCURR,ZCURR)
 WITH U1A AMPS IN THE X-DIRECTION AND
 U1Z AMPS IN THE Z-DIRECTION.

IMPLICIT COMPLEX (C,U)
 REAL COS
 COMPLEX KAPPA
 COMMON/PI/ PI,EPH0,CJ
 COMMON/UBS/ UB2,TH2,PHI2
 COMMON/PATCH/ K0,TH0,PHI0
 COMMON/PATCH/ XCURR,YCURR,ZCURR
 COMMON/PARAM/ AK1,KAPPA,C10

Z2=K0*COS(TH0)+ZCURR
 X2=K0*SIN(TH0)*COS(PHI0)-XCURR
 Y2=K0*SIN(TH0)*SIN(PHI0)-YCURR
 K2=SQRT(X2**2+Y2**2+Z2**2)
 TH2=ACOS(Z2/K2)
 PHI2=PHI0
 IF (X2.NE.0.0) PHI2=ATAN2(Y2,X2)

CALL SGN1(U1,U2,U3)
 K1=SQRT(X2**2+Y2**2+(Z2-2*ZCURR)**2)
 U61=CEXP(-CJ*AK1*K2)/(4.*PI*K1)
 U62=CEXP(-CJ*AK1*K2)/(4.*PI*K2)
 U1A=U1A+C10*(U61-U62+U2)
 U1Z=U1Z+C10*(U61-U62+U1)+U1A*C10*U3
 U61=AK1**2*(COS(TH0)*COS(PHI0)*U1A-SIN(TH0)*U1Z)
 U62=AK1**2*(COS(TH0)*COS(PHI0)*U1A-SIN(TH0)*U1Z)
 RETURN
 END


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00100 SUBROUTINE SKUM1(CP0V1Z,CP0H1X,CP0H1Z)
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00120C
00130C
00140C
00150C
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00980C
00990C
01000C

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SUBROUTINE SKUM1(CP0V1Z,CP0H1X,CP0H1Z)
 COMPUTING THE 1ST TERM OF THE ASYMPTOTIC EXPANSION
 IMPLICIT COMPLEX(C,U)
 REAL COS,C
 COMPLEX KAPPA,U
 COMMON/PI/PI,EP50,CJ
 COMMON/UBS/UBS,TH,PHI
 COMMON/PARAM/AK1,KAPPA,CU
 C=C*EXP(-CJ*AK1*H)/(4.*PI*H)
 C=COS(TH)
 S=SIN(TH)
 U=CSUM1(KAPPA-S**2)
 CP0V1Z=2.*KAPPA*C/(KAPPA*C+U)*CG
 CP0H1X=2.*C/(C+U)*CG
 CP0H1Z=2.*COS(PHI)*S*C*(C+U)/(KAPPA*C+U)*CG
 RETURN
 END

```

00100 SUBROUTINE SKUM2(CP0V1Z,CP0H1X,CP0H1Z)
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00140C
00150C
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00170C
00180C
00190C
00200C
00210C
00220C
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00980C
00990C
01000C

```

SUBROUTINE SKUM2(CP0V1Z,CP0H1X,CP0H1Z)
 COMPUTING THE 2ND TERM IN THE ASYMPTOTIC EXPANSION
 IMPLICIT COMPLEX(C,U)
 REAL COS,C
 COMPLEX KAPPA,U,A1,A2,A3,A4,B1,B2
 COMMON/PI/PI,EP50,CJ
 COMMON/UBS/UBS,TH,PHI
 COMMON/PARAM/AK1,KAPPA,CU
 C2=C*EXP(-CJ*AK1*H)/(4.*PI*AK1*H**2)
 C=COS(TH)
 S=SIN(TH)
 U=CSUM1(KAPPA-S**2)
 A1=KAPPA*U+C
 A2=(S**4+2.*KAPPA*S**2+KAPPA)/U**3
 A3=2.*S**2/U**2+(3.*C-2./C)/U-2.
 A4=S**2/C/U**3+S**2/U**2+2.*C*(3.*C-1./C)/U-6.
 B1=KAPPA*U+C
 B2=C+U
 CP0V1Z=CJ*KAPPA*C*(-2./B1+(A1*(5.*C-2./C)+A2+KAPPA*C)/
 +B1**2+2.*S**2*A1**2/B1**3)*C2
 CP0H1X=CJ*C*(A3/B2+(A2+C)/B2**2)*CG2
 CP0H1Z=COS(PHI)*CJ*S*C*(C+U)*CG2
 CP0H1Z=CP0H1Z*(A4/B1+(A1*(5.*C-2./C)+A2+KAPPA*C)/
 +B1**2+2.*S**2*A1**2/B1**3)
 RETURN
 END

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00100 SUBROUTINE SI(DELTA,P,X,Z)
00110C
00120C   INTEGRATION OF THE FREE SPACE GREENS FUNCTION
00130C   FROM X=DEL/2 TO X+DEL/2, X IS MEASURED ALONG THE
00140C   SOURCE AND P IS THE LATERAL DISPLACEMENT FROM THE
00150C   SOURCE POINT.
00160C
00170C   COMPLEX CMPLX,CEXP,Z,KAPPA
00180C   COMMON/PARAM/ AK,KAPPA,C10
00190C   MEAL 11,12,13,14
00200C   P1=5,14,592654
00210C   ALDELTA/2
00220C   RESUMT(P**2+X**2)
00230C   IF (K=0) GO TO 100
00240C   SI=SUMT(P**2+(X+AL)**2)
00250C   S2=SUMT(P**2+(X-AL)**2)
00260C   I1=ALUG((X+AL+S1)/(X-AL+S2))
00270C   I2=2*AL
00280C   I3=3*(AL+X)*51+5*(AL-X)*52+5*P**2*11
00290C   I4=2*AL*P**2+1.73*(2*AL**3+6*AL*X**2)
00300C   I5=1-(0.1)*AK*(12-K*11)-5*AK**2*(13-2*K*12+K**3*11)
00310C   Z=Z+(I1-I5)/AK**3/6*(14-3*K*13+3*K*12-H**3*11)
00320C   Z=Z+CEXP*(-(0.1)*AK*H)/(4.*PI)
00330C   RETURN
00340C   100 CONTINUE
00350C   AUM=AL/K
00360C   XUM=X/K
00370C   A1=12+1./48*AUM**2*(-1.+3*AUM**2)
00380C   A2=12+1./48*AUM**4*(3.-30*AUM**2+35*AUM**4)
00390C   A3=11/6*AUM*(-1.+3*AUM**2)
00400C   A4=11/48*AUM**3*(3.-30*AUM**2+35*AUM**4)
00410C   A5=-1./6*AUM**2-1./48*AUM**2*(1.-12*AUM**2+15*AUM**4)
00420C   A6=1./60*AUM*(3*AUM**2-5*AUM**4)
00430C   A7=1./120*AUM**4
00440C   Z=Z+(0.1)*AK*AL*A1+(AK*AL)**2*A2
00450C   Z=Z+(0.1)*AK*AL**3*A3+(AK*AL)**4*A4
00460C   Z=Z+2.*AUM*CEXP(-(0.1)*AK*H)/(4.*PI)
00470C   RETURN
00480C   END

```

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00100 SUBROUTINE SPIAPP(CP0V1Z,CP0M1X,CP0M1Z)
00200C
00300C      COMPUTE THE VECTOR POTENTIALS VIA THE APPROXIMATE TECHNIQUE
00400C
00500C
00600C      IMPLICIT COMPLEX(C,U)
00700C      COMPLEX KAPPA
00800C      REAL COS
00900C      COMMON/PI/ PI, EPSZ, CJ
01000C      COMMON/UBS/ K2, IM2, PH12
01100C      COMMON/PARAM/ AK1, KAPPA, C10
01200C      Z2=K2*COS(IM2)  $  KMU2=K2*SIN(IM2)
01300C      STORER=K2  $  STORET=IM2
01400C      CALL SUBP1(UG,UGZ,UGZ2,UGXZ,UGX,0,0)
01500C      CP0M1X=-2./AK1**2/(1.-KAPPA)*-(CJ*AK1*CSUM1(KAPPA)*UGZ+UGZ2)
01600C      CONST=AK1/CSUM1(KAPPA)
01700C      K2=PI5/AK1
01800C      IF(K2.GT.K2P) GO TO 100
01900C      Z2=PI5/K2P**2-KMU2**2)  $  K2=K2P
02000C      CALL SUBP1(UGP,UGZP,UGZ2P,UGXP,UGX,0,0)
02100C      999  FORMAT(/6G14,6)
02200C      K2=STORER  $  IM2=STORET
02300C      CALL SUB(Z2P,Z2,UGP,UG,UGXZP,CONST,USZ,0)
02400C      CALL SUB(Z2P,Z2,UGP,UG,UGXZP,CONST,USZX,1)
02500C      CP0V1Z=USZ
02600C      CP0M1Z=1./AK1**2/KAPPA*(-2.*UGXZ-CJ*CONST*(1.+KAPPA)*USZX)
02700C      RETURN
02800C      100  CALL SHCM1(CP0V1Z,U2,CP0M1Z)
02900C      RETURN
03000C      END
03100C
03200C
03300C      COMPLEX FUNCTION UGARGV(Z2P)
03400C      IMPLICIT COMPLEX(C,U)
03500C      COMPLEX KAPPA
03600C      REAL COS
03700C      COMMON/PI/ PI, EPSZ, CJ
03800C      COMMON/UBS/ K2, IM2, PH12
03900C      COMMON/PARAM/ AK1, KAPPA, C10
04000C      STORER=K2  $  STORET=IM2  $  KMU2=K2*SIN(IM2)
04100C      CONST=AK1/CSUM1(KAPPA)
04200C      K2=SUB1(Z2P**2-KMU2**2)
04300C      CALL SUBP1(UG,UGZ,UGZ2,UGXZ,UGX,0,0)
04400C      UGARGV=UGXZ*CONST*(-CJ*CONST*Z2P)
04500C      K2=STORER  $  IM2=STORET
04600C      RETURN
04700C      END
04800C
04900C
05000C      COMPLEX FUNCTION UGARGH(Z2P)
05100C      IMPLICIT COMPLEX(C,U)
05200C      COMPLEX KAPPA
05300C      REAL COS
05400C      COMMON/PI/ PI, EPSZ, CJ
05500C      COMMON/UBS/ K2, IM2, PH12
05600C      COMMON/PARAM/ AK1, KAPPA, C10
05700C      STORER=K2  $  STORET=IM2  $  KMU2=K2*SIN(IM2)
05800C      CONST=AK1/CSUM1(KAPPA)
05900C      K2=SUB1(Z2P**2-KMU2**2)
06000C      CALL SUBP1(UG,UGZ,UGZ2,UGXZ,UGX,0,0)
06100C      UGARGH=UGXZ*CONST*(-CJ*CONST*Z2P)
06200C      K2=STORER  $  IM2=STORET
06300C      RETURN
06400C      END
06500C
06600C      COMPLEX FUNCTION UGARGH(Z2P)
06700C      IMPLICIT COMPLEX(C,U)

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06300 COMPLEX KAPPA
06400 MEAL COS
06500 COMMON/PI/ PI,EP50,CJ
06600 COMMON/UBS/ M2,IM2,PM12
06700 COMMON/PANAM/ AK1,KAPPA,C10
06800 STOKEMK2 $ STOKET=IM2 $ MMU2=K2*SIN(IM2)
06900 LUNSTAK1/COUT(KAPPA)
07000 MESSUMT(ZP=2*MMU2**2) $ IM2=ATAN(MMU2/ZP)
07100 CALL SUGPI(U1,U2,U3,U4,UBA00)
07200 UGARGH=UBA*CEXP(-CJ*CONST*Z2P)
07300 M2=STURM $ IM2=STURM
07400 RETURN
07500 END
07600 C
07700 C
07800 C
07900 SUBROUTINE SUGPI(UG,UGZ,UGZ2,UGXZ,UGX,IV,IM)
08000 IMPLICIT COMPLEX (C,U)
08100 COMPLEX KAPPA
08200 MEAL COS
08300 COMMON/PI/ PI,EP50,CJ
08400 COMMON/UBS/ M2,IM2,PM12
08500 COMMON/PANAM/ AK1,KAPPA,C10
08600 UGZCEXP(-CJ*AK1*IM2)/(4.*PI*IM2)
08700 IF (IV.EQ.1) RETURN
08800 X2=M2*SIN(IM2)*COS(PM12)
08900 UR1=UM(1)
09000 UGAX=CEMU1*UG RETURN
09100 IF (IM.EQ.1)
09200 Z2=X2*CO$ (IM2)
09300 UGZ2=Z2*UM1*UG
09400 UR2=UM(2)
09500 UGZ2=(UR1*Z2**2*UM2)*UG+Z2*UM1*UGZ
09600 UGZ2=X2*Z2*UM2*UG+X2*UM1*UGZ
09700 RETURN
09800 END
09900 C
10000 C
10100 C
10200 COMPLEX FUNCTION UM(I)
10300 IMPLICIT COMPLEX (C,U)
10400 COMPLEX KAPPA
10500 COMMON/PI/ PI,EP50,CJ
10600 COMMON/UBS/ M2,IM2,PM12
10700 COMMON/PANAM/ AK1,KAPPA,C10
10800 A1=2*PI
10900 UM=1/M2 *A1,I
11000 J1=2*PI*2
11100 IF (J1.GT.1) A1=A1-J1
11200 J2=2*PI*3
11300 IF (J2.GT.1) A1=A1-J2
11400 IF (J3.GT.1) A1=A1-J3

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11500 URG(1)=1*(K1+CJ*AK1**2**K2)/K2**2*(2*1)
11600 RETURN
11700 END
11800
11900
12000 SUBROUTINE SUB(Z2P,Z2,UGP,U0,UGXZP,CONST,US,IX)
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163000 PH12=ATAN2(Y2-DEL,X2)
164000 CALL SMCN1(U1,U2,U3)
165000 IF(ABS(Z2-Z2P).GE.1.E-12) CALL UUG12(Z2P,Z2,UGANGH,USIDYM)
166000 USP=(U1UY-U1UYM)/2./DEL
167000 IF(ABS(Z2-Z2P).GE.1.E-12) USINT=(USIDY-USIDYM)/2./DEL
168000 K2=STOREN $ PH12=STOREP
169000 GO TO 200
170000
171000 $1W IF(1.E-12) GO TO 520
172000 KMO2U=SUMT(X2+DEL)**2+Y2**2)
173000 K2=SUMT(KMO2U**2+Z2P**2) $ TH2=ATAN(KMO2U/Z2P)
174000 PH12=ATAN2(Y2+0.000001*DEL,X2+DEL)
175000 CALL SMCN1(U1,U2,U3DX)
176000 IF(ABS(Z2-Z2P).GE.1.E-12) CALL UUG12(Z2P,Z2,UGANGH,USIDX)
177000 K2=SUMT(KMO2U**2+Z2P**2) $ TH2=ATAN(KMO2U/Z2P)
178000 PH12=ATAN2(Y2+0.000001*DEL,X2-DEL)
179000 CALL SMCN1(U1,U2,U3DXM)
180000 IF(ABS(Z2-Z2P).GE.1.E-12) CALL UUG12(Z2P,Z2,UGANGH,USIOXM)
181000 US=(USDX-U3DXM)/2./DEL
182000 IF(ABS(Z2-Z2P).GE.1.E-12) USINT=(USIOX-USIOXM)/2./DEL
183000 USP=CJ/CUNST/(1.+KAPPA)*(2.*UGXZP+AK1**2*KAPPA*U3)
184000 K2=STOREN $ PH12=STOREP
185000 GO TO 200
186000
187000 $1W IF(1.E-12) RETURN
188000 KMO2U=SUMT(X2**2+(Y2+DEL)**2)
189000 K2=SUMT(KMO2U**2+Z2P**2) $ TH2=ATAN(KMO2U/Z2P)
190000 PH12=ATAN2(Y2+DEL,X2)
191000 CALL SMCN1(U1,U2,U3UY)
192000 IF(ABS(Z2-Z2P).GE.1.E-12) CALL UUG12(Z2P,Z2,UGANGH,USIOY)
193000 KMO2U=SUMT(X2**2+(Y2-DEL)**2) $ TH2=ATAN(KMO2U/Z2P)
194000 PH12=ATAN2(Y2-DEL,X2)
195000 CALL SMCN1(U1,U2,U3UYM)
196000 IF(ABS(Z2-Z2P).GE.1.E-12) CALL UUG12(Z2P,Z2,UGANGH,USIDYM)
197000 US=(USDY-U3UYM)/2./DEL
198000 IF(ABS(Z2-Z2P).GE.1.E-12) USINT=(USIOY-USIDYM)/2./DEL
199000 USP=CJ/CUNST/(1.+KAPPA)*(2.*UGXZP+AK1**2*KAPPA*U3)
200000 K2=STOREN $ PH12=STOREP
201000 GO TO 200
202000
203000
204000 END
205000

```



```

000100 SUBROUTINE SESAPP(UEX,UEY,UEZ,CIV,CIM)
000105C
000110C
000115C
000120C
000125C
000130C
000135C
000140C
000145C
000150C
000155C
000160C
000165C
000170C
000175C
000180C
000185C
000190C
000195C
000200C
000205C
000210C
000215C
000220C
000225C
000230C
000235C
000240C
000245C
000250C
000255C
000260C
000265C
000270C
000275C
000280C
000285C
000290C
000295C
000300C
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000310C
000315C
000320C
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000365C
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000375C
000380C
000385C
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000455C
000460C
000465C
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000475C
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000485C
000490C
000495C
000500C
000505C
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000545C
000550C
000555C
000560C
000565C
000570C
000575C
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000600C
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000625C
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000635C
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000645C
000650C
000655C
000660C
000665C
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000675C
000680C
000685C
000690C
000695C
000700C
000705C
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000930C
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000940C
000945C
000950C
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000970C
000975C
000980C
000985C
000990C
000995C
001000C

      COMPUTE THE THREE COMPLEX SCATTERED E-FIELD COMPONENTS IN THE
      (X,Y,Z) COORDINATE SYSTEM USING THE APPROXIMATE PROCEDURE
      DEVELOPED IN CHAPTER 4.

      IMPLICIT COMPLEX(C,U)
      COMPLEX KAPPA,UDG(17),UDGP(17)
      REAL CUS
      COMMON/PI/ PI,EPS0,CJ
      COMMON/UBS/ K2,TH2,PHI2
      COMMON/PAKAM/ AK1,KAPPA,C10
      ZK2=CUS*(TH2) $ KMU2=K2*SIN(TH2)
      STOMER=K2 $ STOMET=TH2
      CONST=AK1/COSHT(KAPPA)
      CALL S0GES(UDG)
      K2B=15/AK1
      IF(K2B>K2P) GO TO 100
      ZP=SGHT(K2P**2-KMU2**2)
      K2=K2P $ TH2=ATAN(KMU2/ZP)
      CALL S0GES(UDGP)
      GO TO 110
100 ZP=Z2
110 UG=105
115 UDG(1)=UDG(1)
120 CONTINUE
      K2=STOMER $ TH2=STOMET
      UG=UDGP(1)
      UGXP=UDGP(2)
      UGYP=UDGP(3)
      UGZP=UDGP(4)
      UGXZP=UDGP(5)
      UGZ2P=UDGP(6)
      UGY2P=UDGP(7)
      UGXZ2P=UDGP(8)
      UGYZ2P=UDGP(9)
      UGXZ2P=UDGP(10)
      UGXZ2P=UDGP(11)
      UGXZ2P=UDGP(12)
      UGXZ2P=UDGP(13)
      UGXZ2P=UDGP(14)
      UGXZ2P=UDGP(15)
      UGXZ2P=UDGP(16)
      UGXZ2P=UDGP(17)
      UGXZ2P=UDGP(18)
      UGXZ2P=UDGP(19)
      UGXZ2P=UDGP(20)
      UGXZ2P=UDGP(21)
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      UGXZ2P=UDGP(114)
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      UGXZ2P=UDGP(157)
      UGXZ2P=UDGP(158)
      UGXZ2P=UDGP(159)
      UGXZ2P=UDGP(160)
      UGXZ2P=UDGP(161)
      UGXZ2P=
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03100  L0=-1
03200  UP1X=C0*U0+C1*U0Z+C2*U0Z2
03300  UP1XZ2=C0*UGX2+C1*UGXZ2+C2*UGXZ2Z
03400  UEX=UEX+CIM*(AK1**2*UP1X+UP1XX2)
03500  UP1XXY=C0*UGXY+C1*UGXYZ+C2*XYZ2
03600  UEY=UEY+CIM*UP1XXY
03700  UP1XXZ=C0*UGXZ+C1*UGXZ2+C2*UGXZ3

05800  UEZ=UEZ+CIM*UP1XXZ
05900  C
06000  SUB CONTINUE
06100  CALL SUB(Z2P,Z2,UGP,UGXZP,CONST,US,M)
06200  CALL SUB(Z2P,Z2,UGXP,UGXZP,CONST,USX,1)
06300  CALL SUB(Z2P,Z2,UGYP,UGXZP,CONST,USY,2)
06400  CALL SUB(Z2P,Z2,UGXVP,UGXZP,CONST,USXZ,3)
06500  USZ=CJ*CONST*US+2.*UGZ
06600  USZ2=CJ*CONST*USZ+2.*UGZ2
06700  USXZ2=CJ*CONST*USX+2.*UGXZ
06800  USY2=CJ*CONST*USY+2.*UGY2
06900  USXZ2=CJ*CONST*USXZ+2.*UGXZ2
07000  USXYZ=CJ*CONST*USXY+2.*UGXYZ
07100  C10=C1V
07200  C11=-C1M*UJ*CONST*(KAPPA+1.)/(AK1**2*KAPPA)
07300  C12=-C1M*UJ*(AK1**2*KAPPA)
07400  C13=-C1V
07500  UP1Z2=C10*US+2.*USX+C11*USXZ+C12*UGXZ+C13*UGZ
07600  UP1Z2=C10*USY2+C11*USYZ+C12*UGXYZ+C13*UGYZ
07700  UP1Z2=C10*USXZ+C11*USXZ2+C12*UGXZ2+C13*UGXZ
07800  UEY=UEY+UP1Z2
07900  UEY=UEY+UP1YZ
08000  UEZ=UEZ+AK1**2*UP1Z+UP1YZ2
08100  RETURN
08200  END
08300  C
08400  SUBROUTINE SOGES(UBG)
08500  IMPLICIT COMPLEX(C,U)
08600  DIMENSION UBG(17)
08700  REAL COS
08800  COMPLEX KAPPA
08900  COMMON/PI/PI,PI30,CJ
09000  COMMON/UPS/UP1,UP2,PHI2
09100  COMMON/PARA/PA1,PA2,PA3,CIM
09200  UBG(1)=UB=COS(PI2)/(4*PI*PA2)
09300  PHO2=PA2*ASIN(TH2)
09400  X2=PHO2*COS(PI2)
09500  PH1=UR(1)
09600  UBG(2)=UBX=X2*UR1*UB
09700  UBG(3)=UBY=Y2*UR1*UB

```

```

1600  UG(4)=UGZ+Z2*UK1*UG
1700  UK2=UK(2)
1800  UG(5)=UGX2*(UK1+X2**2*UK2)*UG+X2*UK1*UGX
1900  UG(6)=UGZ2*(UK1+Y2**2*UK2)*UG+Y2*UK1*UGY
2000  UG(7)=UGX1*(UK2+X2*UK2*UG+X2*UK1*UGY
2100  UG(8)=UGX2*(UK2+Y2*UK2*UG+Y2*UK1*UGZ
2200  UG(9)=UGY1*(UK2+Z2*UK2*UG+Z2*UK1*UGZ
2300  UK3=UK(3)
2400  UGZ3=(UK2+X2*UK2+Z2**2*UK3)*UG+2*(UK1+Z2**2*UK2)*UGZ+
2500  Z2*UK1*UGZ2
2600  UG(10)=UGX2Z2*(UK2+X2**2*UK3)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
2700  UG(11)=UGXZ2*(UK2+Y2**2*UK3)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
2800  UG(12)=UGXZ2*(UK2+Z2**2*UK3)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
2900  UK4=UK(4)
3000  UG(13)=UGX2Z2*(UK2+X2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
3100  UG(14)=UGXZ2*(UK2+Y2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
3200  UG(15)=UGXZ2*(UK2+Z2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
3300  UG(16)=UGX2Z2*(UK2+X2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
3400  UG(17)=UGXZ2*(UK2+Y2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
3500  UG(18)=UGXZ2*(UK2+Z2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
3600  UG(19)=UGX2Z2*(UK2+X2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
3700  UG(20)=UGXZ2*(UK2+Y2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
3800  UG(21)=UGXZ2*(UK2+Z2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
3900  UG(22)=UGX2Z2*(UK2+X2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
4000  UG(23)=UGXZ2*(UK2+Y2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
4100  UG(24)=UGXZ2*(UK2+Z2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
4200  UG(25)=UGX2Z2*(UK2+X2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGX+Z2*UK1*UGX2
4300  UG(26)=UGXZ2*(UK2+Y2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGY+Z2*UK1*UGY2
4400  UG(27)=UGXZ2*(UK2+Z2**2*UK4)*UG+2*(UK2+Z2**2*UK2*UGZ+Z2*UK1*UGZ2
4500  RETURN
4600  END

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001000 SUBROUTINE SUMINT(CPMV1Z,CPWH1X,CPWH1Z,IV,IM)
002000
003000 COMPUTE SOMMERFELD INTEGRALS
004000 IV=0, DO NOT COMPUTE THE VERTICAL INTEGRAL
005000 IM=0, DO NOT COMPUTE THE HORIZONTAL INTEGRALS
006000
007000
008000 IMPLICIT COMPLEX(C,U)
009000 COMPLEX KAPPA,BRV1Z,BRM1X,BRM1Z
010000 COMMON/PI/ PI,EPS0,CJ
011000 COMMON/UBS/ U2,TH2,PHI2
012000 COMMON/PARAM/ AK1,KAPPA,CIU
013000 EXTERNAL VIZANG,MIXANG,MIZANG
014000 CPMV1Z=CPWH1X=CPWH1Z=(0.,0.)
015000 TMAX=3./SQRT(AK1**2)
016000 A=REAL(CSUMT(KAPPA))
017000 B=AIMAG(CSUMT(KAPPA-1.))
018000 TH2MIN=ASIN(1./SQRT(A**2+B**2))-ATAN(B/A)
019000 TH2P=TH2MIN+180./PI
020000 PMINTH05,TMAX,TH2P
021000 IF (TH2.LT.TMAX) THEN
022000   IF (TH2.LT.TH2MIN) THEN
023000     TMAX=TH2MIN
024000     DEL=5.
025000     U=CSUMT(KAPPA+DEL)
026000     UH=CSUMT(KAPPA-DEL)
027000     TH2P=ASIN(1./SQRT(REAL(UH)**2+AIMAG(UH)**2))-ATAN(AIMAG(UH)/REAL(UH))
028000     IF (ABS(TH2-TH2P).LT.1.E-4) GO TO 20
029000     IF ((TH2-TH2P)*IS.GT.0) GO TO 10
030000     DEL=-DEL/2.
031000     GO TO 10
032000   ELSE
033000     BETA1=BETA
034000     T1=SQRT(-SIN(TH2))*AIMAG(UA)+CUS(TH2)*REAL(UH)
035000     PRINT(0,1,BETA1
036000     IF (1.GT.TMAX)
037000     CALL PRINT(BETA1,BRV1Z,BRM1X,BRM1Z,IV,IM)
038000     IF (IV.EQ.0)
039000     CALL DDB12(-TMAX,0,VIZANG,U1)
040000     CALL DDB12(0,T1-MAX(0,VIZANG,U2)
041000     CALL DDB12(T1+0,T1,VIZANG,U2)
042000     CALL DDB12(T1+0,T1,IMAX,VIZANG,U3)
043000     CPMV1Z=U1+U2+U3+BRV1Z
044000     IF (IM.EQ.0) RETURN
045000     CALL DDB12(-TMAX,0,MIXANG,U41)
046000     CALL DDB12(0,T1-MAX(0,MIXANG,U21)
047000     CALL DDB12(T1+0,T1,MIXANG,U42)
048000     CALL DDB12(0,T1-MAX(0,MIXANG,U22)
049000     CALL DDB12(T1+0,T1,IMAX,MIXANG,U43)
050000     CPMIX=U41+U42+U43+BRM1X
051000     CPMH1Z=U21+U22+U23+BRM1Z
052000     RETURN
053000

```



```

05400 100 IF (V.EU.N) GO TO 110
05500 CALL DUG12(-TMAX,N,VIZARG,U1)
05600 CALL DUG12(N,TMAX,VIZARG,U2)
05700 CPUV12=U1+U2
05800 110 IF (M.EU.N) RETURN
05900 CALL DUG12(-TMAX,N,MIXARG,U1)
06000 CALL DUG12(-TMAX,N,MIZARG,U2)
06100 CALL DUG12(N,TMAX,MIXARG,U2)

06200 UVZ=KAPPA/(KAPPA**2*CUSX1**2-BETA**2)*UM*UM0
06300 10 IF (IMX.EU.N) GO TO 20
06400 CALL MNKL0(C1,UM0)
06500 UMX=UM/(KAPPA-1)*UM0
06600 20 IF (IMZ.EU.N) RETURN
06700 CALL MNKL1(C1,UM1)
06800 UMZ=CJ=COS(PH12)*((KAPPA+1.)/(KAPPA**2*CUSX1**2-BETA**2))*SINX1
06900 +CUSX1*UM*UM1
07000 RETURN
07100 END

```


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00100 SUBROUTINE PIMARG(IVZ,IMX,IMZ,I,UVZ,UMX,UMZ)
00200
00300 SUMMEPELO INTEGRANDS
00400
00500 IMPLICIT COMPLEX(C,U)
00600 REAL COS
00700 COMPLEX KAPPA,XI,SINXI
00800 COMMON/PI/ PI,EPSE,CJ
00900 COMMON/UMS/ UMZ,IMZ,PHI2
01000 COMMON/PAKAM/ AKI,KAPPA,CIO
01100
01200 UVZ=UMX*UMZ*(0.7071)
01300 CALL SUBTAXI(T,IMZ,AI)
01400 RMGZ=K2*SIN(IMZ)
01500 SINXI=CSIN(XI)
01600 COSXI=CCOS(XI)
01700 USU=CSORT(KAPPA-A-SINXI**2)
01800 IF (I) UT=UMX*AI*MAG(USU)
01900 C1=AKI*(2.*PI)*CEXP(-CJ*AKI*IMZ)/CSORT(T**2+2.*CJ)
02000 UM=AKI/((2.*PI)*CEXP(CJ*C1)*EXP(-AKI*IMZ*1+I2))
02100 UM=UM*SINXI=CUSXI*CEXP(CJ*C1)*EXP(-AKI*IMZ*1+I2)
02200 IF (IVZ.EU.0) GO TO 10
02300 IF (IMZ.LT.1.E-6) CH0=-2.*CJ/PI*CL0G(AK1*IMZ*SINXI)
02400 IF (IMZ.GT.1.E-6) CALL MNKL0(C1,CH0)
02500 UVZ=KAPPA*UM*CH0/(KAPPA+CUSXI+USU)
02600 UM IF (IMX.EU.0) CH0=-2.*CJ/PI*CL0G(AK1*IMZ*SINXI)
02700 IF (IMZ.LT.1.E-6) CH0=-2.*CJ/PI*CL0G(AK1*IMZ*SINXI)
02800 IF (IMZ.GT.1.E-6) CALL MNKL0(C1,CH0)
02900 UMX=UM*CH0/(CUSXI+USU)
03000 20 IF (IMZ.EU.0) RETURN
03100 IF (IMZ.LT.1.E-6) RETURN
03200 CALL MNKL1(C1,CH1)
03300 UMZ=-CJ*COS(PHI2)*UM*CH1*SINXI*(CUSXI-USU)/(KAPPA+CUSXI+USU)
03400 RETURN
03500 END
03600
03700
03800 COMPLEX FUNCTION V1ZARG(T)
03900 IMPLICIT COMPLEX(V)
04000 CALL PIMARG(1,0,0,I,V1ZARG,V2,V3)
04100 RETURN
04200
04300 END
04400
04500
04600 COMPLEX FUNCTION MIXARG(T)
04700 IMPLICIT COMPLEX(H)
04800 CALL PIMARG(0,1,0,I,M1,M1XARG,M3)
04900 RETURN
05000
05100 END

```

```

053000C
054000C COMPLEX FUNCTION MIZARG(T)
055000C IMPLICIT COMPLEX(N)
056000C CALL PMARG(0,0,1,1,M1,M2,MIZARG)
057000C RETURN
058000C
059000C END
060000C
061000C
062000C SUBROUTINE DOPXA1(T,IM2,X1)
063000C COMPLEX CMPLX,AL
064000C SUI=SUM1{(I=1,4)} /2.
065000C ALPHA=((-I+2+SU1))/2.
066000C BETA=((-I+2+SU1))/2.
067000C U1=ALUS(ALPHA)SUMT(MEIA+2-1,1)
068000C U2=ALUG(BETA)SUMT(MEIA+2-1,1)
069000C XI=CMPLX(U1,U2)*SIGN(1,1)+IM2
070000C RETURN
071000C
072000C END

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```

001000 SUBROUTINE DPRINT(BETA1,BRV1Z,BRM1X,BRM1Z,IV,IM)
002000
003000
004000
005000
006000
007000
008000
009000
010000
011000
012000
013000
014000
015000
016000
017000
018000
019000
020000
021000
022000
023000
024000
025000
026000
027000
028000
029000
030000
031000
032000
033000
034000
035000
036000
037000
038000
039000
040000
041000
042000
043000
044000
045000
046000
047000
048000
049000
050000
051000
052000
053000
    COMPUTE BRANCH CUT CONTRIBUTIONS
    IV=0 , DO NOT COMPUTE THE VERTICAL CASE
    IM=0 , DO NOT COMPUTE THE HORIZONTAL CASE

    COMPLEX BRV1Z,BRM1X,BRM1Z
    EXTERNAL HANGVZ,HANGHX,HANGHZ
    IF(IV.EQ.0) GO TO 10
    CALL DGG12(0,BETA1,HANGVZ,BRV1Z)
    IF(IM.EQ.0) RETURN
    CALL DGG12(0,BETA1,HANGHX,BRM1X)
    CALL DGG12(0,BETA1,HANGHZ,BRM1Z)
    RETURN
    END

    COMPLEX FUNCTION HANGVZ(BETA)
    IMPLICIT COMPLEX (B)
    REAL BETA
    CALL BRANG(1,0,0,BETA,HANGVZ,B2,B3)
    RETURN
    END

    COMPLEX FUNCTION HANGHX(BETA)
    IMPLICIT COMPLEX (B)
    REAL BETA
    CALL BRANG(0,1,0,BETA,H1,HANGHX,B3)
    RETURN
    END

    COMPLEX FUNCTION HANGHZ(BETA)
    IMPLICIT COMPLEX (B)
    REAL BETA
    CALL BRANG(0,0,1,BETA,B1,B2,HANGHZ)
    RETURN
    END

    SUBROUTINE BRANG(IVZ,IMX,IMZ,BETA,UVZ,UMX,UMZ)
    IMPLICIT COMPLEX (C,U)
    REAL CUS
    COMPLEX KAPPA,SINXI
    COMMON/PI/ PI,PI2,PI12
    COMMON/UBS/ K2,IM2,PH12

```

```

05400 CUMH/MANAM/ AK1,KAPPA,C10
05500 CUS1=-CJ*CSUM1(KAPPA-1,-BETA**2)
05600 SIN1=CSUM1(KAPPA-BETA**2)
05700 KMUSM2=SIN(1M2) $ Z2=M2*CU5(1M2)
05800 UMBAK1/(2*PI*CUJ)*BETA**2*CEXP(-CJ*AK1*Z2*CU5X1)
05900 C1BAK1=KMUS2*SINX1
06000 IF(LV).EQ.0) GO TO 10
06100 CALL MKKL2(C1,UM0)

06200 CALL DGG12(W1,IMAX,MIZAKU,UZ2)
06300 CPMH1XU1+U12
06400 CPMH1ZU1+U22
06500 RETURN
06600 END

```

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```

001000C-----SUBROUTINE XINVT(N,NX,A,D,IEMM)
002000C-----
003000C-----PARAMETERS HAVE THE FOLLOWING MEANINGS
004000C-----A = N BY N REAL SYMMETRIC MATRIX TO BE INVERTED, ON OUTPUT A
005000C-----CONTAINS A INVERSE.
006000C-----D = DETERMINANT OF A.
007000C-----
008000C-----IMPLICIT COMPLEX(A-M,U-Z)
009000C-----
010000C-----INITIALIZE DETERMINANT
011000C-----DETA(1)=1
012000C-----COMPUTE D(1)=(-1)
013000C-----A(1,1)=1/A(1,1)
014000C-----DO 10 I=2,N
015000C-----IM1=I-1
016000C-----DO 20 K=1,IM1
017000C-----GAMMA(K)=0
018000C-----COMPUTE BV=0
019000C-----DO 10 KK=1,IM1
020000C-----10 GAMMA(K)=A(K,KK)*A(1,KK)+GAMMA(K)
021000C-----20 GAMMA(K)=GAMMA(K)
022000C-----ALPHA=0
023000C-----COMPUTE ALPHA
024000C-----DO 30 KK=1,IM1
025000C-----30 ALPHA=A(1,1)+ALPHA
026000C-----ALPHA=A(1,1)+ALPHA
027000C-----IF ALPHA=0 THEN GO TO ERROR RETURN
028000C-----IF (CAHS(ALPHA).LE.1.0E-15)GO TO 60
029000C-----Y=1/(ALPHA)
030000C-----DO 60 K=1,IM1
031000C-----STORE -BV*Y/ALPHA
032000C-----A(1,K)=GAMMA(K)*Y
033000C-----A(K,1)=A(1,K)
034000C-----DO 40 KK=1,K
035000C-----STORE BV+BV*Y*Y/ALPHA
036000C-----IF (A EQ. 1)GO TO 60
037000C-----IF (A EQ. 1)GO TO 60
038000C-----KMI=K-1
039000C-----DO 50 KK=1,KMI
040000C-----STORE -(BV*Y/ALPHA)*
041000C-----50 A(KK,K)=A(K,KK)
042000C-----60 CONTINUE
043000C-----STORE 1/ALPHA
044000C-----A(1,1)=Y
045000C-----UPDATE DETERMINANT
046000C-----70 D=ALPHA*D
047000C-----RETURN
048000C-----
049000C-----
050000C-----
051000C-----
052000C-----END

```



```

SUBROUTINE HREAL(Z,M1)
      NARGEL FUNCTION, 1ST UNDER, SECOND TYPE
      USAGE: INPUT COMPLEX*16 Z
      USAGE: OUTPUT COMPLEX*16 M1
      TO CALCULATE J0 AND Y0 HMKL USES AN ANALYTIC CONTINUATION
      OF THE POLYNOMIAL FITS IN ABRAMOWITZ AND STEGUN PG 369

      COMPLEX A1,Z,M1,Y1,J1
      REAL PI
      PI=3.141592654
      A1=(0.E0,1.E0)
      IF (CABS(Z).LE.12.)GOTO10
      GOTO 20
10  CONTINUE
      CALL Y1(Z,Y1,J1)
      M1=J1-A1*41
      GOTO 199
20  M1=CSUM(2./(PI*Z))*CEXP(-1.*A1*(Z-3.*PI/4.))=(1.+15./(128.*Z+
      2)-A1*3./(8.*Z))
      RETURN
44  END
      SUBROUTINE J1(Z,J1)

      BESSEL FUNCTION, 1ST ORDER
      USAGE: INPUT COMPLEX*16 Z
      USAGE: OUTPUT COMPLEX*16 J1
      ANALYTICAL CONTINUATION INTO THE FIRST QUADRANT OF THE
      POLYNOMIAL FIT IN ABRAMOWITZ AND STEGUN PG 369

      COMPLEX J1,J1AS,J1AL,Z,Z1,A1
      REAL PI,AP,REAL,A1MAG
      PI=3.141592654
      A1=0.E0,1.E0
      Z1=CMPLX{ABS(REAL(Z)),ABS(A1MAG(Z))}
      AP=ATAN2(A1MAG(Z),REAL(Z))
      IF (CABS(Z).LE.5.0)J1=J1AS(Z1)
      IF (CABS(Z).GT.5.0)J1=J1AL(Z1)
      IF (AP.GT.PI/2)AND.AP.LT.PI)J1=CEXP(A1*PI)*CONJG(J1)
      IF (AP.LE.-1.*PI/2)AND.AP.GT.-1.*PI)J1=-1.*PI)*CEXP(A1*PI)*J1
      IF (AP.LT.0)AND.AP.GT.-1.*PI)J1=CONJG(J1)
      RETURN
      END
      COMPLEX FUNCTION J1AS(Z)
      COMPLEX Z,W
      W=Z/3
      J1AS=2*(.5-.56249985*W+.21493573*W**4+.05954289*W**6+.
      +.00445514*W**8+.00031761*W**10+.00001109*W**12)
      RETURN
      END

```

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006400 COMPLEX FUNCTION JIAL(Z)
006500 COMPLEX Z,M,F1,FM
006600 M=3./Z
006700 F1=.79700456+.00000150*PI+.01659667*PI**2+.00017105*PI**3+.
006800+.00024451*PI**4+.00113653*PI**5+.00020033*PI**6
006900 FM=Z-2+.35619449+.12499612*PI+.00005650*PI**2+.00637879*PI**3+.
007000+.00014540*PI**4+.00074024*PI**5+.00029166*PI**6
007100 JIAL=F1*CCOS(FM)/COUNT(Z)
007200 RETURN
007300 END
007400 SUBROUTINE YI(Z,Y1,J1)
007500
007600 NEUMANN FUNCTION, 1ST UNDER, SUBROUTINE J1 IS CALLED
007700 USAGE INPUT COMPLEX*16 Z
007800 ANALYTIC CONTINUATION INTO THE FIRST QUADRANT OF THE
008000 POLYNOMIAL FIT FOUND IN ABRAHONWITZ AND STEGUN PG 369
008100
008200 COMPLEX Y1,YIAS,YIMAG,Z,Z1,A1,J1
008300 REAL PI,AP,REAL,AIMAG
008400 PI=3.141592654
008500 A1=(Z-2)/PI
008600 Z1=COMPLEX(A1,0)
008700 CALL J1(Z1,J1)
008800 A1=ATAN2(AIMAG(Z),REAL(Z)),ABS(AIMAG(Z))
008900 IF(CABS(Z).LE.3.0) Y1=YIAS{Z1,J1}
009000 IF(CABS(Z).GT.3.0) Y1=YIAL{Z1}
009100 IF(AP.GT.PI/2. .AND. AP.LT.PI) Y1=CEXP(A1*PI)*(CONJG(Y1)+2.*A1*
009200+.CONJG(J1))
009300 IF(AP.LE.-1.*PI/2. .AND. AP.GT.-1.*PI) Y1=CEXP(A1*PI)*(Y1-2.*A1*J1)
009400 IF(AP.LT.0. .AND. AP.GE.-1.*PI/2. .Y1=CONJG(Y1)}
009500 IF(AP.GT.PI/2. .AND. AP.LT.PI) J1=CONJG(J1)}
009600 IF(AP.LT.-PI/2. .AND. AP.GE.-PI) J1=J1
009700 IF(AP.LT.0. .AND. AP.GE.-PI/2.) J1=CONJG(J1)
009800 RETURN
009900 END
010000 COMPLEX FUNCTION YIAS(Z,J1)
010100 COMPLEX Z,J1,M
010200 REAL PI
010300 PI=3.141592654
010400 M=Z/3
010500 YIAS=(1./Z)*(2./PI)*CCOS(6./Z/2.)*J1-.0500198+.2212091*PI**2+2.
010600+.10027073*PI**4-.3164027*PI**6+.31233951*PI**8-.0400916*PI**10+.
010700+.00020033*PI**12)
010800 RETURN
010900 END
011000 COMPLEX FUNCTION YIAL(Z)
011100 COMPLEX Z,M,FM,F1
011200 M=3./Z
011300 F1=.79700456+.00000150*PI+.01659667*PI**2+.00017105*PI**3+.
011400+.00024451*PI**4+.00113653*PI**5+.00020033*PI**6
011500 FM=Z-2+.35619449+.12499612*PI+.00005650*PI**2+.00637879*PI**3+.
011600+.00014540*PI**4+.00074024*PI**5+.00029166*PI**6

```

[illegible]


```

022200      THUS 64-18539010-04160597*(3/2)-00003954*(3/2)**2+00262575*(3/
022300+Z)**3-000054125*(3/2)**4-000029333*(3/2)**5+000135588*(3/2)**6
022400      UALFPCUSIN(THU)/CSUMT(Z)
022500      RETURN
022600      END

```

```

001000 SUBROUTINE D0G12(XL,XU,FCI,Y)
001100      INTEGRATE COMPLEX FUNCTION FCT FROM XL TO XU
001200      USING 12-POINT GAUSS QUADRUPTURE FORMULA,
001300      EXACT FOR POLYNOMIALS UP TO DEGREE 23.
001400
001500      COMPLEX Y, FCT
001600      EXTERNAL FCT
001700
001800      A=5*(XU+XL)
001900      B=XU-XL
002000      C=.49070031712335905E00
002100      Y=.25567666193255914E-1*(FCT(A+C)+FCT(A-C))
002200      C=.452050662810523743E00
002300      Y=Y+.53469662997659215E-1*(FCT(A+C)+FCT(A-C))
002400      C=.38495133709715234E00
002500      Y=Y+.8003916427167311E-1*(FCT(A+C)+FCT(A-C))
002600      C=.29365897714330672E00
002700      Y=Y+.10150371330155296E0*(FCT(A+C)+FCT(A-C))
002800      C=.1839157494990010E00
002900      Y=Y+.11074626026917740E0*(FCT(A+C)+FCT(A-C))
003000      C=.02610704255734450E-1
003100      Y=Y+.12457352290070139E0*(FCT(A+C)+FCT(A-C))
003200      RETURN
003300      END

```

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